Vehicle and Crew Scheduling with Flexible Timetable

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1 Introduction

In this article we propose a new model for the simultaneous vehicle and crew scheduling problem occuring in the planning of urban mass transit systems. Our model incorporates the possibility of *trip shifting* in a given range in order to achieve a better solution.

Instead of defining the time windows for each trip individually, we define so-called *flexible groups* which are simply subsets of trips. We call a timetable which consists of flexible groups *flexible timetable*. Trips in the same flexible groups are shifted together.

The purpose of creating flexible timetable is twofold. In one hand, this could decrease the number of decision variables in the model. In the other hand, loss of service quality can be avoided: i) It keeps constant time headway between the trips in the same flexible group. ii) The waiting times of passengers does not change, if the trips of the lines whose connection are important are in the same flexible groups.

Each flexible group g has a time window $[d^g_{\min}, d^g_{\max}]$. The starting time of the trips in the same flexible group g should be changed with the same $t \in [d^g_{\min}, d^g_{\max}]$ value.

The structure of the paper is as follows: In section 2 the new model is presented. A heuristic solution method is described in section 3. Finally, in section 4 the computational result on smaller artifical instances is presented.

2 The Model

Our model is an extension to the model proposed by Haase *et al.* [3]. Let w be the set of trips, indexed by w. Each trip is divided into tasks (d-trip), which have to be performed by drivers. Let V represent the set of tasks, indexed by v. The set of flexible groups is notated by G, indexed by g. The set of trips is partitioned by flexible groups. W_g is the set of trips in the flexible group $g \in G$. Followed by the definition, $W = \bigcup_{g \in G} W_g$ and $W_{g_1} \bigcap W_{g_2} = \oslash, g_1 \neq g_2, g_1 \in G, g_2 \in G$. Similarly, $V = \bigcup_{g \in G} V_g$ and $V_{g_1} \bigcap V_{g_2} = \oslash, g_1 \neq g_2, g_1 \in G, g_2 \in G$. Let H represent the set

of the depot leaving time, indexed by h. Let U be the set of duty-types, indexed by u. The set of feasible duties are $\Omega^u, u \in U$, indexed by ρ.

The following binary parameters used in the model: $e^{\rho}_{v,t}$ is 1, if task v shifted by t time units is covered by duty ρ, otherwise 0. $f^{\rho}_{w,t}$ is 1, if duty ρ contains a driving movement ending at the start station of trip w shifted by t time units, otherwise 0. $g^{\rho}_{w,t}$ is 1, if duty ρ contains a driving movement starting at the end station of trip w shifted by t time units, otherwise 0. Let q^{ρ}_h equal to 1, if duty ρ contains a driving movement starting before time point $h \in H$, and ending after h, otherwise 0.

The binary variable $\theta^u_\rho, \rho \in \Omega^u, u \in U$ represents whether duty ρ is performed. The integer variable B is the minimal number of buses required to cover the schedule. The binary variable $X_{g,t}$ is a time-indexed variable, it is equal to one if the flexible group g shifted by t time units ($t \in [d^g_{\min}, d^g_{\max}]$).

The model of vehicle and crew scheduling with flexible timetable is the following:

$$\min c_B B + \sum_{u \in U} \sum_{\rho \in \Omega^u} c_\rho \theta^u_\rho \tag{1}$$

$$\sum_{u \in U} \sum_{\rho \in \Omega^u} e^{\rho}_{v,t} \theta^u_\rho = X_{g,t}, \qquad \forall g \in G, \forall v \in V_g, \forall t \in [d^g_{\min}, d^g_{\max}] \tag{2}$$

$$\sum_{u \in U} \sum_{\rho \in \Omega^u} f^{\rho}_{w,t} \theta^u_\rho = X_{g,t}, \qquad \forall g \in G, \forall w \in W_g, \forall t \in [d^g_{\min}, d^g_{\max}] \tag{3}$$

$$\sum_{u \in U} \sum_{\rho \in \Omega^u} g^{\rho}_{w,t} \theta^u_\rho = X_{g,t}, \qquad \forall g \in G, \forall w \in W_g, \forall t \in [d^g_{\min}, d^g_{\max}] \tag{4}$$

$$\sum_{u \in U} \sum_{\rho \in \Omega^u} q^{\rho}_h \theta^u_\rho \leq B, \qquad \forall h \in H \tag{5}$$

$$\sum_{t \in [d^g_{\min}, d^g_{\max}]} X_{g,t} = 1, \qquad \forall g \in G \tag{6}$$

$$X_{g,t}, \theta^u_\rho \text{ are binary}, \qquad B \text{ is integer.} \tag{7}$$

The objective function (1) minimizes first the number of buses (here c_B is a large enough number), then the crew cost. (2) represent the task-covering equations. Task v shifted by t time units has to be covered if and only if the corresponding flexible group g is shifted with the same t value ($X_{g,t} = 1$). (3) and (4) are the so-called bus flow conservation equations. There should be exactly one driving movement ending at the start station of trip w shifted by t time units, and one starting at the end station of trip w shifted by t time units, if and only if the corresponding flexible group g is shifted with the same t value. (5) are the bus counting inequalities. (6) force that each flexible group is shifted by exactly one time value.

3 A Solution Approach

The number of feasible duties $|\Omega|$ is very large, direct enumeration of them yields an untreatable large linear problem. A column generation approach is more appropriate. Even with column generation, finding the optimal solution requires unacceptable running time. Because of this, we made the effort to find a heuristic method for solving this problem. Nevertheless, this method is also based on column generation. In the following subsections the main ideas of the method are described.