A Sampling Procedure for Real-Life Rich Vehicle Routing Problems

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Summary. In this paper we address a rich variant of the vehicle routing problem which occurs in real-life applications. Among other aspects we take into consideration time windows, simultaneous delivery and pick-up at customer locations, multiple use of vehicles, and timely allocation of vehicles to loading bays at the depot. In order to solve practical instances of the resulting real-life rich vehicle routing problem, efficient methods are required. For this reason, we present a sampling procedure, which is a multi-start algorithm that executes in each call a substantial extension of the well-known savings algorithm. Using a set of suitable benchmark instances, we assess the performance of the proposed sampling procedure.

1 Introduction

Most current problems faced by logistics operators have been studied for years, e.g. the vehicle routing problem (VRP), which is initially considered by Dantzig and Ramser [2]. The VRP consists of determining routes for vehicles of known capacity which are to operate from a single depot to supply a set of customers with known locations and known demands for a certain commodity. Routes for the vehicles are computed such that an objective function is minimized, e.g. the total distance traveled or the number of vehicles used. The problem has attracted a lot of attention in the academic literature, because it appears in a large number of practical situations and due to its complex structure, it is of theoretical interest. Many methods have been proposed for solving the VRP, either optimally or approximatively; we refer to [4] for a recent survey. In the past few years, the research community has turned to more complex, and rich variants of the VRP that occur in real-life applications. The family of those VRP variants is often identified as rich vehicle routing problems (RVRP). In the following we address a real-life RVRP, considering numerous constraints that are often found in reality (see e.g. [5]):

- heterogenous vehicles,
- a total working time for vehicles from departure to arrival at the depot,
- time windows for customers during which delivery and pick-up can occur,
- a time window for the depot,
- simultaneous delivery and pick-up at customer locations,

- multiple use of vehicles throughout the planning horizon, and
- timely allocation of vehicles to loading bays at the depot.

It should be noted that not all of these constraints may apply and therefore we present a sampling procedure which is able to solve this RVRP as well as generalizations.

2 Sampling Procedure

Due to the complexity of the problem at hand and the necessity to generate solutions in an appropriate time period, we use a *sampling procedure* to solve the problem approximatively. Our sampling procedure (cf. Section 2.1 – 2.4) is a multi-start algorithm that executes in each call a substantial extension of the savings algorithm proposed by Clarke and Wright [1].

The RVRP may be described as a graph theoretic problem. Let $G = (V,A)$ be a complete graph, where $V = \{0,...,n\}$ is the vertex set and A is the arc set. Vertices $i = 1,...,n$ correspond to the customers, whereas vertex 0 represents the depot. Two nonnegative weights d_{ij} and t_{ij} are associated with each arc $\langle i,j \rangle \in A$ which represent the distance and the travel time from vertex i to j, respectively. We assume that the distance and travel time matrices are symmetric and the triangle equation is satisfied; i.e. $d_{ik} \leq d_{ij} + d_{jk}$ for all $i,j,k \in V$. Each customer i is associated with a demand $d_i \geq 0$ that should be delivered and a demand $p_i \geq 0$ that should be picked-up. The service time $s_i > 0$ is the time which is necessary for the loading and unloading activities at customer i. A set of heterogenous vehicles is available at the depot and c_{max} is the capacity of the largest vehicle. To ensure that the feasible region is not empty, we assume that $d_i, p_i \leq c_{max}$ for $i = 1,...,n$. The vehicles have a total working time T_{max} which may not be exceeded by the duration dur_t of any tour t. The service at each customer i must start within an associated time window $[a_i,b_i]$, $a_i \leq b_i$. The time window $[a_0,b_0]$ represents the earliest possible departure from the depot and the latest possible arrival at the depot. We assume that each vehicle may perform at most two tours during the planning horizon.

2.1 Initialization

We start the sampling procedure with a simple solution that contains single customer tours $(0,i,0)$ for each customer $i = 1,...,n$. For each tour $t = (0,i,0)$ we determine a time window $[a(t),b(t)]$, where $a(t)$ is the earliest arrival time at the first customer of tour t and $b(t)$ is the latest arrival time at the first customer of t. Additionally, we define $w(t)$ as the waiting time and $c(t)$ as the core time of t, i.e. the sum of travel times and service times. For $t = (0,i,0)$ we obtain $a(t) = a_i, b(t) = b_i, w(t) = 0,$ and $c(t) = s_i$. Let ES_t, LS_t be the earliest and latest start time of tour $t = (0,i,0)$ and l_t the time which is necessary for loading activities at the depot. Then we are able to calculate $ES_t = \max\{a_0, a(t) - t_{0i} - l_t\}$ and $LS_t = \min\{b(t) - t_{0i} - l_t + dur_t, b_0\} - dur_t$ for all tours t. In the initial solution each customer is served individually by a separate vehicle. Therefore, we assign the single customer tours to the existing vehicles, and if necessary we introduce fictitious vehicles. Later on (cf. Section 2.2), all non-required fictitious vehicles would be eliminated. When two routes $(0,...,i,0)$ and $(0,j,...,0)$ can feasibly be merged into a combined route $(0,...,i,j,...,0)$, a distance saving $s_{ij} = d_{i0} + d_{j0} - d_{ij}$ for $i,j = 1,...,n; i \neq j$ is generated. We calculate all those savings and order them in a non increasing fashion.