Preference Sensitivity Analyses for Multi-Attribute Decision Support

Valentin Bertsch, Jutta Geldermann, and Otto Rentz

Institute for Industrial Production (IIP), University of Karlsruhe (TH), Germany
valentin.bertsch@wiwi.uni-karlsruhe.de

Summary. Contributing to transparency and traceability of decision making processes and taking into account the preferences of the decision makers, multi-criteria decision analysis (MCDA) is suitable to bring together knowledge from different disciplines and fields of expertise. The modelling of the decision makers' preferences is a crucial part of any multi-criteria analysis. In multi-attribute value theory (MAVT), preferential information is modelled by weighting factors (i.e. *inter-criteria* comparisons) and value functions (i.e. *intra-criteria* preferences). However, the uncertainties associated with the determination of these preferential parameters are often underestimated. Thus, the focus of this paper is the description of an approach to explore the impact of simultaneous variations of these subjective parameters. Special attention is paid to the consideration of variations of the value functions' shapes. The aim of the presented methods is to facilitate the process of preference modelling and to comprehensibly visualise and communicate the impact of the preferential uncertainties on the results of the decision analysis.

1 Introduction

A decision making process in practice is subject to various sources of uncertainty. The occurring uncertainties can be classified in many different ways, see for instance [8, 3, 5]. According to their respective source, a distinction can be made between *data uncertainties* (uncertainties of the input data to a model), *parameter uncertainties* (uncertainties related to the model parameters) and *model uncertainties* (uncertainties resulting from the fact that models are ultimately only simplifications/approximations of reality). *Model uncertainties* are difficult to quantify and can also be regarded as inherent to the nature of any model. They are thus not considered in this paper. An approach to model, propagate and visualise *data uncertainties* within a decision support system has for instance been proposed in [5].

In this paper, special attention is paid to the uncertainties associated with the *preference parameters* of a MCDA model. These uncertainties are usually examined by means of "common" sensitivity analyses where the best-known

approach allows to examine the effects of varying a weighting parameter of a MCDA model. A major drawback of this approach is that it is limited to varying one (weighting) parameter at a time. The use of Monte Carlo simulation for the consideration of simultaneous variations of the weights of a decision model is described in [1]. This paper presents an extension of the approach to examine the impact of simultaneous variations of the value functions' shapes and their domains' boundaries. In addition, a combined consideration of the different types of preferential uncertainties and their respective impacts on the results of the analysis is proposed.

2 Value Function Sensitivity Analysis

In MAVT, the overall performance score of an alternative a is usually evaluated as

$$v(a) = \sum_{j=1}^{n} w_j \cdot v_j\left(s_j(a)\right) \tag{1}$$

where $1 \leq j \leq n$ and n denotes the number of considered attributes, $s_j(a)$ denotes the score of alternative a with respect to attribute j, $w = (w_1, ..., w_n)$ ($\sum_{j=0}^{n} w_j = 1$, $w_j \geq 0$ for all j) is the weighting vector and $v_j(x)$ (with $x = s_j(a)$) is the value function to evaluate the performance of alternative a with respect to attribute j. For the latter, the common one-parameter representation for exponential value functions will be used in the following (see also Fig. 1):

$$v_j(x) = \begin{cases} \dfrac{1-e^{-\frac{\Delta_j x}{\rho_j}}}{1-e^{-\frac{x_{max}^j - x_{min}^j}{\rho_j}}}, & \rho_j \neq \pm\infty \\[2ex] \dfrac{\Delta_j x}{x_{max}^j - x_{min}^j}, & otherwise \end{cases} \tag{2}$$

with

$$\Delta_j x = \begin{cases} x - x_{min}^j & \text{for increasing preferences}\,, \\ x_{max}^j - x & \text{for decreasing preferences}\,. \end{cases} \tag{3}$$

2.1 Varying the Value Functions' Shapes

In the following, the impact of varying the parameters ρ_j (i.e. varying the shape(s) of the value function(s)) shall be analysed. For this, let $\rho = (\rho_1, ..., \rho_n)$ denote the vector of the value functions' shape parameters. In order to explore the effect of simultaneously varying the value functions' shapes, we propose to assign an interval $I_j(\rho)$ to each attribute j by determining a lower (ρ_j^l) and an upper (ρ_j^u) bound instead of assigning a discrete value ρ_j: