Dimensionality Reduction in Multiobjective Optimization: The Minimum Objective Subset Problem

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Summary. The number of objectives in a multiobjective optimization problem strongly influences both the performance of generating methods and the decision making process in general. On the one hand, with more objectives, more incomparable solutions can arise, the number of which affects the generating method's performance. On the other hand, the more objectives are involved the more complex is the choice of an appropriate solution for a (human) decision maker. In this context, the question arises whether all objectives are actually necessary and whether some of the objectives may be omitted; this question in turn is closely linked to the fundamental issue of conflicting and non-conflicting optimization criteria. Besides a general definition of conflicts between objective sets, we here introduce the $\mathcal{NP}$-hard problem of computing a minimum subset of objectives without losing information (`MOSS`). Furthermore, we present for `MOSS` both an approximation algorithm with optimum approximation ratio and an exact algorithm which works well for small input instances. We conclude with experimental results for a random problem and the multiobjective 0/1-knapsack problem.

1 Motivation

It is indisputable, that a high number of criteria in a multiobjective optimization problem [5, 10] can cause additional difficulties compared to a low-dimensional problem. The number of incomparable solutions can raise [7], the (human) decision making process becomes harder and generating methods may require substantially more computational resources[1] when more criteria are involved in the problem formulation. Consequently, the question arises whether it is possible to omit some of the objectives without changing the characteristics of the underlying problem. Furthermore, one may ask under which conditions such an objective reduction is feasible and how a minimum set of objectives can be computed.

[1] For example, when based on the $\mathcal{S}$-metric [15].

These questions have gained only little attention in the literature so far. Transferred to the multiobjective optimization setting, closely related research topics as principal component analysis [9] or dimension theory [14] aim at determining a (minimum) set of *arbitrary* objective functions that preserves (most of) the problem characteristics; however, here we are interested in determining a minimum subset of *original* objectives that maintains the order on the search space. Known attempts dealing with the latter do not preserve the dominance structure [4] or are not suitable for black-box scenarios [1]. Furthermore, there are a few studies that investigate the relationships between objectives in terms of conflicting and nonconflicting optimization criteria. The definitions of conflicting objectives are based on trade-offs within the Pareto optimal set [1, 3, 8] or on the number of incomparable solutions [11, 13]. Conflicts are either defined between objective pairs [1, 8, 11, 13] or as a property of the entire objective set [3]. However, these definitions are not sufficient to indicate whether objectives can be omitted or not; examples can be constructed, cf. [2], where all objectives are conflicting according to [3, 11, 13], but one among the given objectives can be removed while preserving the search space order.

Assuming that the decision making process is postponed after the search process, this paper addresses two open issues for a given set of (trade-off) solutions: (i) deriving general conditions under which certain objectives may be omitted and (ii) computing a minimum subset of objectives needed to preserve the problem structure between the given solutions. In particular, we

- propose a generalized notion of objective conflicts which comprises the definitions of Deb [3], Tan et al. [13], and Purshouse and Fleming [11],
- specify on this basis a necessary and sufficient condition under which objectives can be omitted,
- introduce the $\mathcal{NP}$-hard problem of minimum objective subsets (`MOSS`),
- provide both an approximation algorithm and an exact algorithm for `MOSS`,
- validate our approach in additional experiments by comparing the algorithms and investigating the influence of the number of objectives and the search space size.

2 A Notion of Objective Conflicts

Without loss of generality, in this paper we consider a minimization problem with k objective functions $f_i : X \to \mathbb{R}$, $1 \leq i \leq k$, where the vector function $f := (f_1, \ldots, f_k)$ maps each solution $\mathbf{x} \in X$ to an objective vector $f(\mathbf{x}) \in \mathbb{R}^k$. Furthermore, we assume that the underlying dominance structure is given by the weak Pareto dominance relation[2] which is defined as follows: $\preceq_{\mathcal{F}'} :=$

[2] Any other preorder rel like the ε-dominance relation can be used as well if $rel_{\mathcal{F}} = \bigcap_{1 \leq i \leq k} rel_i$ holds for any set $\mathcal{F} = \{f_1, \ldots, f_k\}$ of k objective functions. The proof of the equivalence is simple for $\preceq_{\mathcal{F}}$ and can be found in [2].