Comparison of Stochastic- and Guaranteed-Service Approaches to Safety Stock Optimization in Supply Chains

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Summary. Contributions to multi-echelon safety stock optimization can be classified as being either of the stochastic- or guaranteed-service framework. Though both approaches have been used in inventory theory for a long time, there is still a lack of a detailed comparison. In this paper, such a comparison is presented. Differences in the materials flow are outlined and insights into the performance of both approaches are gained from a simulation study.

1 Introduction

In supply chains, safety stock is a prominent means to deal with demand uncertainty. Developing computationally tractable approaches for determining the appropriate amount of safety stock in fairly general systems is a complex task. Models that deal with the optimization of stock levels in a multi-echelon context fall into two major categories (see, e.g., [3]): stochastic-service approach (SSA) and guaranteed-service approach (GSA) models. The approaches differ in terms of the underlying materials flow concept and the resulting service time characteristics. The service time is the time it takes until an order placed by a stockpoint is shipped to the stockpoint by its predecessor. In the SSA, safety stock is regarded as the only means to deal with supply and demand uncertainty. Therefore, the service time depends on the material availability at the supplying stockpoint. Although a stockpoint might have a deterministic processing time, its entire replenishment time is stochastic due to potential stockouts at upstream stockpoints. The fundamental work by [1] shows that the optimal inventory control rule for such a system is an echelon order-up-to policy. In GSA models, which build on the work by [5], other uncertainty countermeasures besides safety stock exist, so-called operating flexibility, which comprise, e.g., overtime or accelerated production. Therefore, each stockpoint quotes a service time to its successor that it can always satisfy. After this deterministic time span, the successor receives all requested materials and can start processing. While both approaches have been used in inventory theory for a long time, so far no detailed comparison has been drawn between them.

The purpose of this paper is to compare the two modelling approaches described in Sect. 2 and gain general insights into their relative performance by analyzing the simplest version of a supply chain, a two-stage serial one. Section 3 reports the results of a simulation study and points out the main findings. In particular, the service time guarantee of the GSA is explicitly taken into account in the simulation model. Thus, the effect of having a single means to deal with demand uncertainty versus multiple means can be investigated.

2 Multi-Echelon Safety Stock Optimization Approaches

In the SSA and the GSA, each stockpoint i in the supply chain performs a certain processing function, e.g., a step in a manufacturing or transportation process, and is a potential location for holding safety stock after the process has finished. The numbering of stockpoints i is done in increasing order from the most upstream to the most downstream one, $i = 1,2,...,n$. The processing time λ_i is assumed to be deterministic and an integer multiple of a base period. No capacity constraints exist. Customer demand is assumed to be stationary and independent across non-overlapping intervals with mean μ and standard deviation σ. f_λ denotes the λ-period demand probability density. Unsatisfied customer demands are backordered. Without loss of generality, production coefficients are set equal to 1. Moreover, all stockpoints operate a periodic-review base-/echelon-stock policy with a common review period.

2.1 Stochastic-Service Approach

In the SSA, stock insufficiencies at the supplying stockpoint delay the delivery of the stockout quantity until new material becomes available from incoming orders. Consequently, the replenishment time L_i at each stockpoint i, $L_i = ST_{i-1} + \lambda_i$, is stochastic as it consists of the stochastic service time of the predecessor of i, ST_{i-1}, and its own processing time, λ_i. To achieve a predefined external service level, sufficient stock has to be held to cope with demand variability during the replenishment time. Due to the echelon order-up-to policy, each stockpoint takes into account all available amount of stock in the downstream part of the supply chain and orders such that the external service level is achieved. Whereas the external service level is given exogenously, internal service levels are decision variables within the optimization.

Due to the decomposition result derived by [1], optimal echelon order-up-to levels, S_i, can be obtained starting with the final-stage stockpoint (see [6] for details). For the two-stage case, S_i have to be set such that

$$\int_0^{S_2} f_{\lambda_2}(u)\,du = \frac{p+h_1}{p+h_2} \quad \text{and} \quad \int_0^{S_2}\int_0^{S_1-u} f_{\lambda_1}(v)\, f_{\lambda_2}(u)\,dv\,du = \frac{p}{p+h_2} \quad ,$$

where S_i and h_i denote the echelon order-up-to level and holding cost of stockpoint i, respectively. For a given out-of-stock probability $1-\alpha$, a so-called α-service level, the required penalty cost per item short, p, can be determined by $\alpha = p/(p+h_2)$. The local base-stock level of i is given by the difference of echelon order-up-to levels of adjacent stockpoints, $S_i - S_{i+1}$. An exact mathematical solution for more complex