A Branch and Bound Algorithm Based on DC Programming and DCA for Strategic Capacity Planning in Supply Chain Design for a New Market Opportunity

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1 Introduction

Supply chain network are considered as solution for effectively meeting customer requirements such as low cost, high product variety, quality and shorter lead times. The success of a supply chain lies in good strategic and tactical planning and monitoring at the operational level. Strategic planning is long term planning and usually consists in selecting providers and distributors, location and capacity planning of manufacturing/servicing units, among others.

In this paper we only consider the problem of strategic capacity planning proposed in [5, 6]. We consider a three-echelon system : providers, producers and distributors. We assume that all partners under consideration have already met the pre-requisite requirements. All distributors have a definite demand in each period on a given horizon. Providers provide the raw material/semi-finished products to selected producers and these ones fulfill the demand of each distributor. Each provider and producer has its production cost and transportation cost to the next stage. These costs are invariant in time. The transportation costs vary from one pair (provider-producer and producer-distributor) to another. Each provider and each producer has limited production and transportation capacities. The transportation and production capacities can be extended by investing in resource. We assume that investment is only possible at the beginning of the first period. The transportation costs and the production costs are linear functions of quantities. Investment cost is also a linear function of quantity, but with additional fixed cost which does not depend on quantities but only on the entities it is related to. The problem is to select the most economic combination of providers and producers such that they satisfy the demand imposed by all distributors.

This problem is modeled as a mixed-integer program as follows. Let $i \in \{1,2,\ldots,P\}$ be the providers , $j \in \{1,2,\ldots,M\}$ be the producers and $k \in \{1,2,\ldots,D\}$ be the distributors. The demand is deterministic and is known for each distributor on time horizon T. We denote

- p_i : Raw material cost per unit at provider i.
- m_j : Production cost per unit at producer j.
- u_{ij} : Transportation cost per unit from provider i to producer j.
- v_{jk} : Transportation cost per unit from producer j to distributor k.
- R_{ij}, r_{ij} : Available, added transportation capacities from i to j.
- S_{jk}, s_{jk} : Available, added transportation capacities from j to k.
- G_i, g_i : Available, added production capacities for provider i.
- H_j, h_j : Available, added production capacities for producer j.
- d_k^t : Demand of distributor k in period t.

We consider four investments in this model : investment to enhance the capacity of providers, investment to enhance the capacity of producers and investment to enhance the transportation capacity between provider-producer and producer-distributor. So we have the following four investment cost :

- Investment cost for new transportation capacity r_{ij} from provider i to producer j : Cost $= \mathbf{A_{ij}} + \mathbf{a_{ij} r_{ij}}$ if $r_{ij} > 0$ and equal to $\mathbf{0}$ if $r_{ij} = 0$
- Investment cost for new transportation capacity s_{jk} from producer j to distributor k : Cost $= \mathbf{B_{jk}} + \mathbf{b_{jk} s_{jk}}$ if $s_{jk} > 0$ and equal to $\mathbf{0}$ if $s_{jk} = 0$
- Investment cost for provider i to enhance its capacity by g_i : Cost $= \mathbf{E_i} + \mathbf{e_i g_i}$ if $g_i > 0$ and equal to $\mathbf{0}$ if $g_i = 0$
- Investment cost for producer j to enhance its capacity by h_j : Cost $= \mathbf{F_j} + \mathbf{f_j h_j}$ if $h_j > 0$ and equal to $\mathbf{0}$ if $h_j = 0$

Finally, we have the model with the objective function

$$\sum_{t=1}^{T}\sum_{i=1}^{P}\sum_{j=1}^{M} C_t(p_i+u_{ij})x_{ij}^t + \sum_{t=1}^{T}\sum_{j=1}^{M}\sum_{k=1}^{D} C_t(m_j+v_{jk})y_{jk}^t +$$
$$\sum_{i=1}^{P}\sum_{j=1}^{M}(C_1-D_1)(a_{ij}r_{ij}+A_{ij}U_{ij}) + \sum_{j=1}^{M}\sum_{k=1}^{D}(C_1-D_1)(b_{jk}s_{jk}+B_{jk}V_{jk}) +$$
$$\sum_{i=1}^{P}(C_1-D_1)(e_ig_i+E_iW_i) + \sum_{j=1}^{M}(C_1-D_1)(f_jh_j+F_jX_j) \tag{1}$$

where $C_t = (1+\alpha)^{T-t+1}, D_t = (1-\beta)^{T-t+1} \quad \forall t \in \{1,2,\ldots T\}$ with α is the discount rate and β is the depreciation factor.
and the constraints :

$$\sum_{j=1}^{M} x_{ij}^t \le G_i + g_i, \quad x_{ij}^t \le R_{ij} + r_{ij} \quad t \in \overline{1,T}\ , i \in \overline{1,P}\ , j \in \overline{1,M} \tag{2}$$

$$\sum_{k=1}^{D} y_{jk}^t \le H_j + h_j, \quad y_{jk}^t \le S_{jk} + s_{jk} \quad t \in \overline{1,T}\ , j \in \overline{1,M}\ , k \in \overline{1,D} \tag{3}$$