Priority-Rule Methods for Project Scheduling with Work Content Constraints

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Summary. In many practical applications of resource-constrained project scheduling a work content (e.g., in man months) is specified for each activity instead of a fixed duration and fixed resource requirements. In this case, the resource usages of the activities may vary over time. The problem then consists in determining the resource usage of each activity at each point in time such that precedence constraints, resource scarcity, and specific constraints on the evolution over time of resource usages are respected and the project duration is minimized. We present two priority-rule methods for this problem and report on results of an experimental performance analysis for instances with up to 200 activities.

1 Introduction

The resource-constrained project scheduling problem $PS|prec|C_{max}$ consists in determining a start time for each project activity such that precedence and resource constraints are met and the project duration is minimized. It is usually assumed that the activity durations as well as the resource requirements of the activities are fixed. For a review of resource-constrained project scheduling we refer to [5,1,10].

For certain practical applications, however, it is more convenient to specify a work content in resource-time units, e.g., in man-hours, for each activity that has to be accomplished. These applications include aggregate project scheduling where detailed activity information may be unknown or uncertain as well as scheduling of labor-intensive projects, e.g., in research and development. The resource usages of an activity may then vary during its execution. Thereby, specific constraints on the evolution over time of resource usages have to be taken into account (work content constraints). The duration of an activity results implicitly from its evolution of resource usages.

This modification of $PS|prec|C_{max}$ which will be referred to as the project scheduling problem with work content constraints has been treated by [3,2,6,7], as well as [8]. None of these approaches seems to be capable of solving problem instances of practical size taking into account all the work content constraints considered in

this paper. We therefore present two priority-rule methods. The first priority-rule method is based on a serial generation scheme that simultaneously determines the evolution of resource usages and the start time for an activity at a time. The second priority-rule method makes use of a parallel generation scheme which schedules parts of the activities in parallel. To this end, certain decision periods are considered successively. For details on both methods, we refer to [4].

2 Problem Statement

We assume that n non-interruptible project *activities* $1,\dots,n$ have to be scheduled. The set of all activities is denoted by V. Between two activities $i,j \in V$ $(i \neq j)$, there may be a *precedence constraint* $\langle i,j \rangle$ indicating that activity j may start as soon as activity i has been completed. In this case, i (j) is called a *direct predecessor* (*direct successor*) of j (i). If there are precedence constraints $\langle i,\iota_1 \rangle, \langle \iota_1,\iota_2 \rangle, \dots, \langle \iota_m,j \rangle$ with $\iota_1, \iota_2, \dots, \iota_m \in V$ $(m > 0)$, i (j) is called a *indirect predecessor* (*indirect successor*) of j (i).

Several *renewable resources* with limited capacity are needed for the execution of the activities. For each activity $i \in V$, a *work content* is given that has to be processed by one of these renewable resources. This resource will be referred to as the activity-independent *work content resource*. At each point in time during execution of activity i, the number of units of the work content resource required to process i may vary between a given *minimum* and a given *maximum resource usage*. Furthermore, the length of some time interval during which no variation of the usage of work content resource by some activity $i \in V$ takes place may not be smaller than a given number of time units which will be called the *minimum block length*.

With respect to the renewable resources different from the work content resource, we assume that their usage by some activity $i \in V$ is a non-decreasing function of the respective usage of the work content resource. Eventually, we assume that the resource usages of each activity can only be changed at integral points in time.

The *project scheduling problem with work content constraints* then consists in determining an *activity profile*, i.e., an assignment of a usage of the work content resource to each time period, for each activity $i \in V$ such that

(1) the precedence constraints are met,
(2) the resource capacity of neither the work content resource nor any other resource is exceeded during any period,
(3) for each activity, the number of resource-time units processed without interruption equals its work content,
(4) the resource usage of each activity does not fall below the activity's minimum resource usage in any period where the activity is processed,
(5) the resource usage of each activity does not exceed the activity's maximum resource usage in any period where the activity is processed,
(6) for each activity, the minimum number of successive periods without change of the resource usage is not less than the given minimum block length,

and the project duration is minimized. Constraints (3) to (6) are referred to as the *work content constraints*.