Fare Planning for Public Transport

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Summary. In this paper we investigate the *fare planning model* for public transport, which consists in designing a system of fares maximizing the revenue. We discuss a discrete choice model in which passengers choose between different travel alternatives to express the demand as a function of fares. Furthermore, we give a computational example for the city of Potsdam and discuss some theoretical aspects.

1 Introduction

The design and the level of fares influence the passenger volume and consequently the revenue of a public transport system. Therefore, they are an important instrument to improve the profitability of the public transport system or to achieve other goals, e.g., to provide access to public transport for the general public.

Some articles in the literature deal with different approaches to find optimal fares for public transport. Hamacher and Schöbel [6] develop a model for designing fares and zones maximizing the similarity to a given fare system, e.g., a distance dependent one. Kocur and Hendrickson [7] and De Borger, Mayeres, Proost, and Wouters [5] introduce models for maximizing the revenue and the social welfare, respectively, subject to several budget constraints. The majority of the literature on public transport fares, however, discusses only theoretical concepts, e.g. marginal cost pricing (Pedersen [8]) and price elasticities (Curtin [4]).

In this article, we want to investigate a model to compute the fares that optimize the revenue for the public transport. This model is called the *fare planning model*. The main advantage of our approach is the inclusion of the public transport network. This allow us to distinguish different travel routes, e.g. between means of transportation like bus or subway, between slow and fast, short and long routes. Therefore it is possible to design complex and optimal fare systems.

This work is a summary of the master thesis “Mathematische Preisplanung im ÖPNV”, that I wrote at the Zuse Institute Berlin. The advisor was Prof. Dr. Grötschel. Some parts of the master thesis were published in the Operations Research Proceedings 2005 [3], another summary can be found in [2]. The theoretical results in Section 3.2 include some new aspects.

2 General Fare Planning Model

We consider a traffic network whereas the nodes V represent the stations and $D \subseteq V \times V$ is a set of *origin-destination pairs* (OD-pairs or traffic relation). Furthermore, we are given a finite set $\mathcal{C}$ of *travel choices.* A travel choice can be a certain ticket type, e.g., single ticket or monthly ticket. Moreover, it can be a combination of a ticket type and a number of trips, which are performed within a certain time horizon, e.g., 40 trips with a monthly ticket in one month.

Let $p^i_{st} : \mathbb{R}^n \to \mathbb{R}_+$ be the *price function* for travel choice $i \in \mathcal{C}$ and OD-pair $(s,t) \in D$, i.e., it determines the price for the given travel choice and the given OD-pair. The price function depends on a *fare vector* $\mathbf{x} \in \mathbb{R}^n_+$ of $n \in \mathbb{N}$ fare variables $x_1, \ldots, x_n$, which we call *fares* in the following. The *demand functions* $d^i_{st}(\mathbf{x})$ measure the amount of passengers that travel from s to t with travel choice i, depending on the fare system $\mathbf{x}$; they are assumed to be *nonincreasing.*

We denote by $\mathbf{d}_{st}(\mathbf{x})$ the vector of all demand functions associated with OD-pair (s,t) and by $\mathbf{d}(\mathbf{x}) = (\mathbf{d}_{st}(\mathbf{x}))$ the vector of all demand functions. Analogous notation is used for $(p^i_{st}(\mathbf{x}))$. The *revenue* $r(\mathbf{x})$ can then be expressed as

$$r(\mathbf{x}) := \mathbf{p}(\mathbf{x})^{\mathrm{T}}\mathbf{d}(\mathbf{x}) = \sum_{i \in \mathcal{C}} \sum_{(s,t) \in D} p^i_{st}(\mathbf{x}) \cdot d^i_{st}(\mathbf{x}) \,.$$

With this notation our general model for the fare planning problem is:

$$\text{(FPP)} \qquad \begin{aligned} &\max\, \mathbf{p}(\mathbf{x})^{\mathrm{T}}\mathbf{d}(\mathbf{x}) \\ &\;s.t.\ \mathbf{x} \in P \,. \end{aligned} \tag{1}$$

All restrictions on the fare variables are included in the set $P \subseteq \mathbb{R}^n$. Here, one can also include social and political aspects like a minimum level of demand or a maximum level of fares.

In the model (FPP) we assume constant costs and a constant level of service. In further investigations we included costs and maximized the profit, i.e., revenue minus costs. Other possible objectives were considered as well, e.g., maximization of the demand with respect to cost recovery. The goal is to make a first step with (FPP) towards a decision support tool for optimizing fare systems. We show the practicability of (FPP) on a prototype example in Section 3.1.

3 Fare Planing with a Discrete Choice Model

Our model expresses passenger behavior in response to fares by the demand function d^i_{st}. In this section, we use discrete choice models, especially the logit model, to obtain a realistic demand function. Therefore we assume that the passengers have full knowledge of the situation and act rationally with respect to the change of the fares. A thorough exposition of discrete choice analysis and logit models can be found in Ben-Akiva and Lerman [1].

In a discrete choice model for public transport, each passenger chooses among a finite set A of *alternatives* for the travel mode, e.g., single ticket, monthly ticket, bike, car travel, etc.

We consider a time horizon T and assume that a passenger which travels from s to t performs a random number of trips $X_{st} \in \mathbb{Z}_+$ during T, i.e., X_{st} is a discrete