

# Multiobjective Evolutionary Algorithms on Complex Networks

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**Abstract.** Spatially structured populations have been used in evolutionary computation for many years. Somewhat surprisingly, in the multiobjective optimization domain, very few spatial models have been proposed. In this paper, we introduce a new multiobjective evolutionary algorithm on complex networks. Here, the individuals in the evolving population are mapped onto the nodes of alternative complex networks – regular, small-world, scale-free and random. A selection regime based on a non-dominance rating and a crowding mechanism guides the evolutionary trajectory. Our model can be seen as an extension of the standard cellular evolutionary algorithm. However, the dynamical behaviour of the evolving population is constrained by the particular network architecture. An important contribution of this paper is the detailed analysis of the impact that the structural properties of the network – node degree distribution, characteristic path length and clustering coefficient – have on the behaviour of the evolutionary algorithm using benchmark bi-objective problems.

## 1 Introduction

In many real-world search and optimization tasks, we are often confronted with a problem involving several incommensurable and often conflicting objectives. A family of equivalent non-dominated compromises – the *Pareto-optimal set* – represent solutions for this class of problem [1,2]. These solutions are optimal in the wider sense that no other solution in the search space is superior to them when all objectives are considered. The goal of any multiobjective optimization technique is to generate a diverse set of points distributed along the non-dominated front.

Evolutionary algorithms are now an established technique for solving multiobjective optimization problems [1,2] (such algorithms will be referred to as MOEAs). Well known MOEAs include NSGA-II [3], SPEA2 [4] and PAES [5]. Typically, these models evaluate a population of candidate solutions with respect to each objective. Non-dominated solutions are identified and form the mating pool, which then undergoes evolutionary transformations. As the model is iterated, the non-dominated set converges towards the true Pareto-optimal set.

Non-dominated sorting routines, the use of an external archive with appropriate niching and elitism mechanisms are often incorporated into the model to help ensure that the algorithm produces a uniformly distributed non-dominated front at the end of the search.

It has long been recognized that parallelism offers important advantages for evolutionary computation systems [6,7]. However, in the MOEA domain, there has only been a relatively small number of parallel models described as compared with the single objective domain (see [8] for a review). In recent years, there has been an increased interest in the study of complex networks in many areas including communication networks, biological networks and sociology [9,10,11,12]. Recent results reported from evolutionary game theory [13,14] and evolutionary algorithms evolving on both regular and small-world networks for single objective problems [6,15,16], suggest that the topology of the network influences the overall behaviour of the evolving population. In this study, we extend this work into the multiobjective optimization domain.

We present a new MOEA where the individuals of the population are mapped onto the nodes of alternative complex networks – regular lattice, small-world, scale-free and random (see Section 4 for an overview). A key component of the model is that individuals only interact with their local neighbours. Here, the network topology defines the local neighbourhood size. Thus, for different network architectures there will be different local neighbourhood sizes and average path lengths between individuals. The key hypothesis we investigate here, is that the structural characteristics of a given network will influence the quality of solutions generated by a MOEA.

The remainder of the paper is organized as follows: In Section 2, we formally describe multiobjective optimization problems. This is followed by a brief review of parallel evolutionary algorithms in Section 3, with an emphasis on cellular-based models. In Section 4, we describe complex networks, including a description of the particular network architectures used in this study. In Section 5, the new MOEA on complex networks is presented. This is followed by a description of the simulation experiments and results in Section 6. Finally, in Section 7 a discussion of the results is presented in terms of the impact of the underlying network topology and future research directions are identified.

## 2 Multiobjective Optimization

Multi-objective problems are problems that consist of a set of objective functions to be minimized or maximized subject to specified constraints. A multi-objective optimization problem can be stated generally as follows [2]:

$$\begin{aligned}
 &\text{Minimize} && f(x) = [f_1(x), f_2(x), \dots, f_k(x)]^T \\
 &\text{subject to:} && g_i(x) \geq 0, \quad i \in [1, \dots, q] \\
 &&& h_i(x) = 0, \quad i \in [1, \dots, p]
 \end{aligned} \tag{1}$$

where  $x$  is a vector of decision variables,  $g_i$  is an inequality constraint, and  $h_i$  is an equality constraint. A solution is said to dominate another if, for all objectives,