Sequential Approximation Method in Multi-objective Optimization Using Aspiration Level Approach

Yeboon Yun[1], Hirotaka Nakayama[2], and Min Yoon[3]

[1] Kagawa University, Kagawa 761-0396, Japan
yun@eng.kagawa-u.ac.jp
[2] Konan University, Kobe 658-8501, Japan
nakayama@konan-u.ac.jp
[3] Konkuk University, Seoul 143-701, Republic of Korea
myoon515@konkuk.ac.kr

Abstract. One of main issues in multi-objective optimization is to support for choosing a final solution from Pareto frontier which is the set of solution to problem. For generating a part of Pareto optimal solution closest to an aspiration level of decision maker, not the whole set of Pareto optimal solutions, we propose a method which is composed of two steps; i) approximate the form of each objective function by using support vector regression on the basis of some sample data, and ii) generate Pareto frontier to the approximated objective functions based on given the aspiration level. In addition, we suggest to select additional data for approximating sequentially the forms of objective functions by relearning step by step. Finally, the effectiveness of the proposed method will be shown through some numerical examples.

1 Introduction

Many decision making problems are formulated as multi-objective optimization problem so as to satisfy the diverse demands of decision maker. Usually, there does not necessarily exist an optimal solution which minimizes all objective functions simultaneously, because of the trade-off among the objective functions. And then, Pareto optimal solution is introduced, and the set of them in the objective function space is called Pareto frontier. Generally, there exist a number of Pareto optimal solutions, which are considered as the candidates of a final decision making solution. Therefore, it is one of main issues in multi-objective optimization how to obtain Pareto optimal solutions, and how to choose one solution from many Pareto optimal solutions. To the end, the aspiration level methods have been developed. These methods search a decision making solution by processing the following two stages repeatedly: 1) solving auxiliary optimization problem to obtain the closest Pareto optimal solution to a given aspiration level of decision maker, and 2) revising her/his aspiration level by making the trade-off analysis. For the cases with many objective functions, it is difficult to visualize Pareto frontier, and also to depict the trade-off among many objective functions. In this case, the conventional interactive optimization methods

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are useful, although these approaches give one Pareto optimal solution with a single-optimization run. On the other hand, it may be the best way to depict Pareto frontier in the cases with two or three objective functions, since visualizing Pareto frontier helps to grasp trade-off among the objective functions. For that purpose, genetic algorithm (GA) has been applied for solving a multi-objective optimization problem, and multi-objective GA (MOGA) has been shown to be effective for generating Pareto optimal solutions. However, MOGA has some problems as follows; i) it is difficult to treat many objective functions, ii) so many function evaluations are needed in generating the whole Pareto frontier. In particularly, the number of function evaluations is very important when applying MOGA as well as conventional multi-objective optimization methods to the real problems such engineering design problem which have black-box objective functions whose forms are not explicitly known in terms of design variables. Under this circumstance, the value associated with each design variable is given by sampled real/computational experiments such as structural analysis, fluid-mechanical analysis, thermodynamic analysis, and so on. These analyses take long execution time and high cost. Therefore, it is essential to reduce the number of function evaluations as few as possible which is needed in finding an optimal solution.

In multi-objective optimization considering the number of function evaluations, it would rather be practicable to generate a necessary part, not the whole of Pareto frontier. In this research, we propose a new method which is composed of two stages; i) the first stage is to approximate the form of each objective function by using support vector regression on the basis of some sample data, ii) in the second stage using MOGA, we generate Pareto frontier to the approximated objective functions based on a given aspiration level of decision maker. Furthermore, we discuss the way how to select additional data for revising the forms of objective functions by relearning step by step. Finally, we illustrate the effectiveness of proposed method through a numerical example.

2 Support Vector Regression

In this section, to begin with, we introduce support vector regression (SVR), which is a kind of Support vector machine (SVM) for function approximation.

SVM has been recognized as a powerful machine learning technique. SVM was originally developed for pattern classification and later extended to regression, [2], [3], [10], [12]. Therefore, we review briefly SVM for classification.

Let $\mathcal{F}$ be a space of conditional attributes. For binary classification problems, the value of $+1$ or -1 is assigned to each pattern $\boldsymbol{x}_i \in \mathcal{F}$ according to its class $\mathcal{A}$ or $\mathcal{B}$. The aim of machine learning is to predict which class newly observed patterns belong to on the basis of the given training data set $(\boldsymbol{x}_i, y_i)$ $(i = 1, \dots, \ell)$, where $y_i = +1$ or -1. This is performed by finding a discriminant function $h(\boldsymbol{x})$ such that $h(\boldsymbol{x}) \geqq 0$ for $\boldsymbol{x} \in \mathcal{A}$ and $h(\boldsymbol{x}) < 0$ for $\boldsymbol{x} \in \mathcal{B}$. Linear discriminant functions, in particular, can be expressed by a linear form