Heatmap Visualization of Population Based Multi Objective Algorithms

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Abstract. Understanding the results of a multi objective optimization process can be hard. Various visualization methods have been proposed previously, but the only consistently popular one is the 2D or 3D objective scatterplot, which cannot be extended to handle more than 3 objectives. Additionally, the visualization of high dimensional parameter spaces has traditionally been neglected. We propose a new method, based on heatmaps, for the simultaneous visualization of objective and parameter spaces. We demonstrate its application on a simple 3D test function and also apply heatmaps to the analysis of real-world optimization problems. Finally we use the technique to compare the performance of two different multi-objective algorithms.

Keywords: Visualization; Multi-objective optimization; Multi-objective algorithms; Evolutionary algorithms; Real-world applications.

1 Introduction

Visualization of the optimal solutions plays a very important role in multi-objective optimization (MO). In MO with conflicting objectives there is no single optimum, and search methods return a set of solutions from which one must be selected. In order to select the solution, a decision maker usually needs to visualize the discovered solutions in the objective space. This can be done using scatterplots of the objective space if there are only 2 or 3 objectives.

Visualization is also used to show the quality of the solutions. A good set of optimal solutions should contain well-distributed converged solutions along the Pareto front. Almost every new algorithm in MO is tested on several 2- and 3-objective problems, and beside numerical measurements, the obtained solutions are illustrated in objective space plots. This illustration is very valuable for many applications in science and industry as domain experts get information about the whole set of optimal solutions. Also, understanding the algorithm behavior is easier with this view.

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Visualizing the objective space directly is possible for 2 and 3 objective spaces whereas for higher number of objectives, the solutions are only be evaluated by metrics. Metrics and numerical measures hide too much information and only consider the objective space. On the other hand, in almost all of the proposed methods and applications in MO, there is a high (>3) number of parameters which are not being visualized. However, showing the parameter values and visualizing them has a great impact on the decision making process.

Here, we investigate several visualization methods in order to visualize parameters and objective values of a set of optimal solutions. We note previous application of objective plots, Self Organizing Maps (SOM) [2], and Distance and Distribution Charts[1].

We also introduce a new application of pre-existing visualization methods to population based algorithms: Heatmap visualization, which has previously been used mainly for the visualization of biological data. Heatmaps typically make full use of color, but this isn't available in this printed paper. A pre-print with full color figures is available as a technical report[16].

When evaluating visualization methods, it is wise to consider what features of the system we may wish to reveal. The possibilities for population based MOEAs include: the diversity and convergence of the solutions in both objective and parameter space; the relationship between parameter and objective values; A comparison between different runs or algorithms; Identification of clusters of solutions in either the parameter or objective spaces; Dynamically illustrating the progress of an algorithm towards its optimization goals.

This introductory section gives some background on Multi-objective Optimization and previous visualization methods which have been applied to it.

To familiarize the reader with our methods, we make use of a 3-objective test problem defined in Section 2. We then apply heatmaps to this problem in sections 3. Sections 4 applies heatmaps on a real world application in mineralogy. A second application in hydrological modeling shows the use of our methods for the comparison of the behavior of algorithms (Section 5).

1.1 Multi-objective Optimization Problems (MOPs)

A Multi-objective Optimization Problem (MOP) contains several objective functions, which are to be optimized at the same time:

$$\begin{aligned}&\text{minimize}\ \vec{f}(\vec{x}) = (f_1(\vec{x}), \cdots, f_m(\vec{x}))\\&\text{subject to}\quad \vec{e}(\vec{x}) < 0\\&\vec{x} \in S\end{aligned}$$

involving $m \geq 2$ (normally conflicting) objective functions $f_i : \Re^n \rightarrow \Re^m$ that we want to minimize simultaneously. The parameters $\vec{x} = (x_1, \cdots, x_n)$ belong to the feasible region *S*. The feasible region is formed by constraint functions $\vec{e}(\vec{x})$. We call the image of the feasible region feasible objective region. Its elements are called objective vectors and they consist of objective values.