Multi-objective Pole Placement with Evolutionary Algorithms

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Abstract. Multi-Objective Evolutionary Algorithms (MOEA) have been succesfully applied to solve control problems. However, many improvements are still to be accomplished. In this paper a new approach is proposed: the Multi-Objective Pole Placement with Evolutionary Algorithms (MOPPEA). The design method is based upon using complex-valued chromosomes that contain information about closed-loop poles, which are then placed through an output feedback controller. Specific cross-over and mutation operators were implemented in simple but efficient ways. The performance is tested on a mixed multi-objective $\mathcal{H}_2/\mathcal{H}_\infty$ control problem.

Keywords: Multi-objective control; Pole placement; Evolutionary Algorithms.

1 Introduction

Most control design problems can be solved using numerical optimization. Thus, Multi-Objective Evolutionary Algorithms (MOEA) have been successfully applied for this purpose, provided the problem is non-convex in the optimization parameters and cannot be efficiently solved by conventional local optimization algorithms [1]. To illustrate this point, let's review five previous related publications. Note that an excellent survey can be found in [2].

In 1995, Fonseca and Fleming [3] developed an approach to multiple objective and constraint handling with genetic algorithms, with application to control system design. A Multiple Objective Genetic Algorithm (MOGA) was proposed, which is still frequently used in many applications.

In 1995, Whidborne *et al* [4] compared the performance of three search methods. Two were based on hill-climbing techniques: Nelder-Mead Dynamic Min-Max (NMDM) and Moving Boundaries Process (MBP). The third was precisely MOGA. The three were found to be useful for interactive multi-objective controller design. Besides, the author introduced MODCONS: a MATLAB toolbox for Multi-Objective Design of Control Systems.

In 2000, Herreros [5] proposed an algorithm for Multi-objective Robust Control Design (MRCD). It was tested against a Linear Matrix Inequalities (LMI) approach for mixed multi-objective $\mathcal{H}_2/\mathcal{H}_\infty$ control problems. An adaptive search space was proposed, motivated by two reasons: the selection of the initial

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population and the delimitation of the search space. In fact, these are still open problems in the field.

In 2005, Liu and Ishihara [6] discussed the use of multi-objective genetic algorithms and the method of inequalities. The performance of the proposed design method was tested on a special set of benchmark control problems.

In 2006, Molina-Cristobal *et al* [7] compared MOGA against a LMI approach to find the trade-off of a multi-objective $\mathcal{H}_2/\mathcal{H}_\infty$ control problem. The author asserted that MOGA could find an improved Pareto-optimal front compared to the LMI approach.

Despite all this important work, many improvements remain still to be accomplished. This includes elements like parameters tuning, space search adaptation, performance assessment and controller coding. Particularly, regarding the latter, a new approach for solving the design problem is proposed in this work: the Multi-Objective Pole Placement with Evolutionary Algorithms (MOPPEA) technique.

The main idea is using complex-valued chromosomes, containing information about closed-loop poles. This representation allows poles be placed through a classical observer-based feedback controller, based on the information contained within each chromosome. Note that, unlike this representation, usually controllers are coded in terms of real parameters [8]. Specific cross-over and mutation operators were implemented in simple but efficient ways.

The exposition is organized as follows. In section **2**, we formulate the controller design problem. The proposed solution method is described in section **3**. In section **4**, it is applied to solve a mixed $\mathcal{H}_2/\mathcal{H}_\infty$ control problem. Finally, conclusions are drawn in section **5**.

2 Problem Formulation

2.1 Preliminaries

Let $L_2^{p\times m}$ be the set of $p \times m$ matrix functions $f : \mathcal{R}_0^+ \to \mathcal{R}^{p\times m}$ such that:

$$\int_0^{+\infty} trace[f^T(t)f(t)]dt < \infty \tag{1}$$

Let $\mathcal{R}(s)^{p\times m}$ be the set of $p \times m$ rational complex matrix functions $G : \overline{\mathfrak{C}}^+ \to \mathfrak{C}^{p\times m}$ such that:

$$G_{ij}(s) = \frac{b_{nu}s^{nu} + b_{nu-1}s^{nu-1} + .. + b_1 s + b_0}{s^{nd} + a_{nd-1}s^{nu-1} + .. + a_1 s + a_0} \quad i = 1, 2, ..., p \quad j = 1, 2, ..., m \tag{2}$$

and $nd \geq nu$, $\forall G_{ij}(s)$; $a_k, b_h \in \mathcal{R}$, $k = 1, 2, ..., nd$ and $h = 1, 2, ..., nu$.