Combinatorial Optimization of Stochastic Multi-objective Problems: An Application to the Flow-Shop Scheduling Problem

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Abstract. The importance of multi-objective optimization is globably established nowadays. Furthermore, a great part of real-world problems are subject to uncertainties due to, *e.g.*, noisy or approximated fitness function(s), varying parameters or dynamic environments. Moreover, although evolutionary algorithms are commonly used to solve multi-objective problems on the one hand and to solve stochastic problems on the other hand, very few approaches combine simultaneously these two aspects. Thus, flow-shop scheduling problems are generally studied in a single-objective deterministic way whereas they are, by nature, multi-objective and are subjected to a wide range of uncertainties. However, these two features have never been investigated at the same time.

In this paper, we present and adopt a proactive stochastic approach where processing times are represented by random variables. Then, we propose several multi-objective methods that are able to handle any type of probability distribution. Finally, we experiment these methods on a stochastic bi-objective flow-shop problem.

Keywords: multi-objective combinatorial optimization, stochasticity, evolutionnary algorithms, flow-shop, stochastic processing times.

1 Introduction

A large part of concrete optimization problems are subject to uncertainties that have to be taken into account. Therefore, many works relate to optimization in stochastic environments (see [10] for an overview), but very few deal with the multi-objective case where Pareto dominance is used to compare solutions. Thus, Hughes [8] and Teich [18] independently suggested to extend the concept of Pareto dominance in a stochastic way by replacing the rank of a solution by its probability of being dominated; but both studies make an assumption on probability distributions. In [1], another ranking method, based on an average value per objective and on the variance of a set of evaluations, is presented. Likewise, Deb and Gupta [5] proposed to apply standard deterministic multi-objective optimizers using an average value, determined over a sample of objective vectors, for each dimension of the objective space. Finally, Basseur and Zitzler [2]

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recently extended the concept of multi-objective optimization using quality indicators [21] to take stochasticity into account. However, even if existing methods are generally adaptable to the combinatorial case, most of them were only tested on continuous mathematical test functions. Thence, it is not obvious that the performances of these algorithms are similar for combinatorial and continuous problems. Furthermore, a large part of these algorithms exploits problem knowledge that may not be available in real-world applications.

The deterministic indicator-based approach [21] consists in assigning each Pareto set approximation a real value reflecting its quality, using a function I [20]. The goal is then to identify a Pareto set approximation that optimizes I. As a result, I induces a total order into the set of aproximation sets in the objective space, and gives rise to a total order into the corresponding objective vectors. The interest of this perception is that no additional diversity preservation mechanisms are required, the concept of Pareto dominance not being directly used for fitness assignment. To extend this approach to the stochastic case, we must consider that every solution can be associated to an arbitrary probability distribution over the objective space.

In this paper, we propose various models to represent stochasticity for a bi-objective flow-shop scheduling problem. Then, we introduce different ways to handle uncertainty, insisting on the various technical aspects. And, we apply the resulting methods to the concrete case of a flow-shop scheduling problem with stochastic processing times, that have, to our knowledge, never been investigated in a multi-objective form. Each approach has advantages and drawbacks and is adapted from indicator-based optimization.

The paper is organized as follows. In section 2, we formulate a bi-objective flow-shop scheduling problem with stochastic processing times (SFSP). In section 3, we present three different approaches dedicated to stochastic multi-objective optimization and apply them on a SFSP. Section 4 presents experimental results. And finally, the last section draws conclusion and suggests further topics in this research area.

2 A Bi-objective Flow-Shop Scheduling Problem with Stochastic Processing Times

The flow-shop is one of the numerous scheduling problems. It has been widely studied in the literature (see, for example, [6] for a survey). However, the majority of works dedicated to this problem considers it on a deterministic single-criterion form and mainly aims at minimizing the makespan, which is the completion time of the last job. Following the formulation of the deterministic model of a bi-objective flow-shop scheduling problem, this section presents various sources of uncertainty that have to be taken into account and introduces different probability distributions to model stochastic processing times. Note that, although this part focuses on the flow-shop, it can easily be generalizable to other types of problem.