

# Local Search Guided by Path Relinking and Heuristic Bounds

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**Abstract.** In this paper we present three path relinking approaches for solving a bi-objective permutation flowshop problem. The path relinking phase is initialized by optimizing the two objectives using Ant Colony System. The initiating and guiding solutions of path relinking are randomly selected and some of the solutions along the path are intensified using local search. The three approaches differ in their strategy of defining the heuristic bounds for the local search, i.e., each approach allows its solutions to undergo local search under different conditions. These conditions are based on local nadir points. Several test instances are used to investigate the performances of the different approaches. Computational results show that the decision which allows solutions to undergo local search has an influence in the performance of path relinking. We also demonstrate that path relinking generates competitive results compared to the best known solutions of the test instances.

## 1 Introduction

Multiobjective optimization (MO) is a field that has been extensively applied to various disciplines. It has many applications in the areas of science and engineering, medicine, finance, operations research and many others. This is one reason why, over the last decades, many researchers have devoted their resources to developing and improving the theories and methodologies of MO.

MO involves solving problems having more than one objective. For example, in the permutation flowshop scheduling problem, where  $n$  jobs have to be sequentially processed on  $m$  machines, possible objectives are (i) to minimize makespan and (ii) to minimize total tardiness. Given the release date  $r_i$  of job  $i$ , due date  $d_i$ , and processing time  $p_{ij}$  on machine  $j$ , makespan is defined as

$f_1 = \max_i \{C_{i,m}\}$  and total tardiness  $f_2 = \sum_i^n \max(C_{i,m} - d_i, 0)$  where  $C_{i,m}$  is

the completion time of job  $i$  at the last machine  $m$ . In this paper, we try to find job sequences such that the above objectives are accomplished. We call this problem as bi-objective permutation flowshop scheduling problem (BPFSP).

In general, there is no single solution that simultaneously accomplishes the objectives of a bi-objective optimization problem. Hence, the Pareto optimal solutions or sometimes called the set of efficient solutions are considered. We say that a solution  $x$  is an efficient solution if there exists no other feasible solution  $y$  such that  $f_k(y) \leq f_k(x)$ , for  $k = 1, 2$  and  $f_k(y) < f_k(x)$  for some  $k$ . Otherwise, we say that  $x$  is dominated by  $y$  and we denote this by  $y \prec x$ .

BPFSP is an  $\mathcal{NP}$ -hard problem since makespan minimization has been proven  $\mathcal{NP}$ -hard for more than two machines [1]. Furthermore, the minimization of total tardiness for one machine has been proven  $\mathcal{NP}$ -hard as well [2]. Therefore, the use of metaheuristics is appropriate.

Path relinking is a population-based heuristic first developed as an intensification strategy for elite solutions obtained by tabu search or scatter search [3]. A path relinking operation starts by selecting an initiating solution  $I_A$  and a guiding solution  $I_B$  from a set  $\mathcal{G}$  of initial solutions. It then generates a path  $P : I_A - I_1 - I_2 - \dots - I_B$  through a given neighborhood space  $\mathcal{N}_{PR}$  where the distance  $d(I_i, I_B)$  between  $I_i$  and  $I_B$  is decreasing monotonically in  $i$ , i.e.,  $d(I_i, I_B) < d(I_j, I_B) \forall i > j$ . In most cases, there exists a huge number of paths  $P$  to be evaluated. Hence, a path selection mechanism is used to choose the preferred path. An intensification phase or local search may be used to improve the quality of the solutions along the path. Finally, PR requires a strategy for updating the set  $\mathcal{G}$  where the next  $I_A$  and  $I_B$  will be selected from.

In general, PR is composed of the following:

- \* Initial population  $\mathcal{G}$
- \* Neighborhood structure  $\mathcal{N}_{PR}$
- \* Distance measure  $d$
- \* Selection criteria for initiating and guiding solutions
- \* Path selection criteria
- \* Local search for the solutions generated
- \* Update strategy for set  $\mathcal{G}$

In this study, we solve the BPFSP using a PR approach. Our PR uses the ant colony system (ACS) to generate the starting solutions. We will also investigate how the different strategies for computing the heuristic bounds for local search affect the performance of PR. We consider three definitions for our heuristic bounds i.e., for deciding whether local search is applied or not. The three heuristic bounds are defined by local nadir points. A local nadir point corresponds to the worst objectives of two given efficient solutions. Finally we will also demonstrate that our PR approach is competitive with respect to the other existing metaheuristics for BPFSP.

This paper is organized as follows. Section 2 describes the implementation of path relinking in BPFSP. Section 3 presents the numerical results of the study and Sect. 4 provides a short conclusion of the study.