Combining Linear Programming and Multiobjective Evolutionary Computation for Solving a Type of Stochastic Knapsack Problem

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Abstract. In this paper, the design of systems using mechanical or electrical energy-transformation devices is treated as a knapsack problem. Due to the well-known NP-hard complexity of the knapsack problem, a combination of integer linear programming and evolutionary multi-criteria optimization is presented to solve this real problem with promising experimental results.

1 Introduction

Many real-world problems involve the simultaneous optimization of multiple objectives. These objectives often conflict, that is, an optimal performance in one objective implies a low performance in some of the remaining objectives. Therefore, a compromise solution must therefore be reached.

Several mathematical programming methods have been developed to deal with multi-objective problems through different approaches [1]. From a mathematical point of view, a multi-objective problem is solved by finding a set of non-dominated solutions. For large and complex problems, however, there are practical difficulties involved in using mathematical programming methods to obtain this set of Pareto-optimal solutions. In recent years, techniques based on meta-heuristics have been proposed to increase efficiency when solving real multi-objective problems [2,3].

Evolutionary computational approaches are used successfully to solve hard problems. Inspired in natural evolution, evolutionary approaches are considered powerful stochastic search methods [4]. Evolutionary strategies evolve a population of potential solutions in a single run by means of chromosomal encoding representation of solutions and crossing, mutation and selection operators. Approaches of this type are therefore of interest in attemps to solve multi-objective problems. For an extended review [5,6,7,8,9].

The knapsack problem is a well-known NP-hard combinatorial optimization problem [10] that can be formulated as follows: given a number of item types

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with corresponding unit profit p_i and unit weights w_i, and a knapsack with fixed capacity c, determine the number of x_i of each item type that maximizes profit without exceeding the capacity limit. Mathematically:

$$\max \sum_i p_i x_i$$
$$\text{subject to}$$
$$w_i x_i \leq c$$
$$x_i \geq 0$$

Due to its application to many real problems, several meta-heuristic methods are proposed to solve combinatorial problems [4,11,12]. Evolutionary techniques are also applied to multi-objective knapsack problems [13,14].

In this paper, a combination of integer programming and multi-objective optimization is presented to handle a complex real problem in the energy sector. The real problem is related to the design of systems using mechanical or electrical devices. More specifically, the number and type of electrical or mechanical devices required to make up the system must be determined.

Due to the complexity of the real problem, the proposed methodology is based on the *divide and conquer* philosophy. In this way, integer linear programming and evolutionary multi-criteria optimization are combined to solve the real problem.

The paper is organized as follows. The stochastic knapsack real problem and its mathematical formulation is described in Section 2. Section 3 presents the proposed methodology to solve the problem. Section 4 analyses the experimental results. Finally, a brief set of conclusions and suggestions for future work are presented in Section 5.

2 Problem Statement and Mathematical Modelling

2.1 Problem Statement

This work addresses the design of mechanical or electrical equipment to achieve a given target. In this case, the target is related to a transformation process. The system or equipment receives an input and delivers an output. The design task is to determine the number and type of electrical and/or mechanical devices required to build the system. There are different types of devices performing the same task. Each device has a set work capacity, efficiency curve, and cost. The capacity is the highest input the device is able to handle. The efficiency curve describes the output-to-input ratio. Each configuration of the system (i.e. a set of devices) is therefore assessed by its total cost and total efficiency.

The transformation process is performed through time. In this problem, it is assumed that the *total input* I is a random quantity whose probability distribution is known or can be estimated.