

# On the Interactive Resolution of Multi-objective Vehicle Routing Problems

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**Abstract.** The article presents a framework for the resolution of rich vehicle routing problems which are difficult to address with standard optimization techniques. We use local search on the basis on variable neighborhood search for the construction of the solutions, but embed the techniques in a flexible framework that allows the consideration of complex side constraints of the problem such as time windows, multiple depots, heterogeneous fleets, and, in particular, multiple optimization criteria. In order to identify a compromise alternative that meets the requirements of the decision maker, an interactive procedure is integrated in the resolution of the problem, allowing the modification of the preference information articulated by the decision maker. The framework is implemented in a computer system. Results of test runs on multiple depot multi-objective vehicle routing problems with time windows are reported.

**Keywords:** User-guided search, interactive optimization, multi-objective optimization, multi depot vehicle routing problem with time windows, variable neighborhood search.

## 1 Introduction

The vehicle routing problem (VRP) is one of the classical optimization problems known from operations research with numerous applications in real world logistics. In brief, a given set of customers has to be served with vehicles from a depot such that a particular criterion is optimized. The most comprehensive model therefore consists of a complete graph  $G = (V, A)$ , where  $V = \{v_0, v_1, \dots, v_n\}$  denotes a set of vertices and  $A = \{(v_i, v_j) \mid v_i, v_j \in V, i \neq j\}$  denotes the connecting arcs. The depot is represented by  $v_0$ , and  $m$  vehicles are stationed at this location to service the customers  $v_1, \dots, v_n$ . Each customer  $v_i$  demands a nonnegative quantity  $q_i$  of goods and service results in a nonnegative service time  $d_i$ . Traveling on a connecting arc  $(v_i, v_j)$  results in a cost  $c_{ij}$  or travel time  $t_{ij}$ . The most basic vehicle routing problem aims to identify a solution that serves all customers, not exceeding the maximum capacity of the vehicles  $Q_k$  and their maximum travel time  $T_k$  while minimizing the total distances/costs of the routes.

Various extensions have been proposed to this general problem type. Most of them introduce additional constraints to the problem domain such as time windows, defining for each customer  $v_i$  an interval  $[e_i, l_i]$  of service. While arrival before  $e_i$  results in a waiting time, arrival after  $l_i$  is usually considered to be infeasible [1]. In other approaches, the time windows may be violated, leading to a tardy service at some customers. Violations of time windows are either integrated in the overall evaluation of solutions by means of penalty functions [2], or treated as separate objectives in multi-objective approaches [3].

Some problems introduce multiple depots as opposed to only a single depot in the classical case. Along with this sometimes comes the additional decision of open routes, where vehicles do not return to the place they depart from but to some other depot. Also, different types of vehicles may be considered, leading to a heterogeneous fleet in terms of the abilities of the vehicles.

Unfortunately, most problems of this domain are  $\mathcal{NP}$ -hard. As a result, heuristics and more recently metaheuristics have been developed with increasing success [4,5,6]. In order to improve known results, more and more refined techniques have been proposed that are able to solve, or at least approximate very closely, a large number of established benchmark instances [7]. It has to be mentioned however, that with the increasing specialization of techniques a decrease in generality of the resolution approaches follows. As a result, heuristic optimization frameworks such as HotFrame [8], EasyLocal++ [9] or ParadisEO [10] try to address this issue by providing generic libraries for the resolution of optimization problems.

While the optimality criterion of minimizing the total traveled distances is the most common, more recent approaches recognize the vehicle routing problem as a multi-objective optimization problem [11,3,12,13,14]. Important objectives besides the minimization of the total traveled distances are in particular the minimization of the number of vehicles in use, the minimization of the total tardiness of the orders, and the equal balancing of the routes. Following these objectives, it is desired to obtain solutions that provide a high quality of delivery service while minimizing the resulting costs. As many objectives are however of conflicting nature, not a single solution exists that optimizes all relevant criteria simultaneously. Instead, the overall problem lies in identifying the set of Pareto-optimal solutions  $P$  and selecting a most-preferred solution  $x^* \in P$ . In this context, three different general strategies of solving multi-objective optimization problems can be implemented:

1. *A priori* approaches reduce the multi-objective problem to a single-objective surrogate problem by formulating and maximizing a utility function. The advantage of this approach can be seen in its simplicity given the possibility to specify the precise utility function of the decision maker. The concept may however not be used if the decision maker is not able to state his/her preferences in the required way.
2. *A posteriori* approaches first identify the Pareto set  $P$ , and then allow the decision maker to select a most-preferred solution  $x^* \in P$ . The main advantage of this resolution principle is, that the computation of the optimal