Substitute Distance Assignments in NSGA-II for Handling Many-Objective Optimization Problems

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Abstract. Many-objective optimization refers to optimization problems with a number of objectives considerably larger than two or three. In this paper, a study on the performance of the Fast Elitist Non-dominated Sorting Genetic Algorithm (NSGA-II) for handling such many-objective optimization problems is presented. In its basic form, the algorithm is not well suited for the handling of a larger number of objectives. The main reason for this is the decreasing probability of having Pareto-dominated solutions in the initial external population. To overcome this problem, substitute distance assignment schemes are proposed that can replace the crowding distance assignment, which is normally used in NSGA-II. These distances are based on measurement procedures for the highest degree, to which a solution is nearly Pareto-dominated by any other solution: like the number of smaller objectives, the magnitude of all smaller or larger objectives, or a multi-criterion derived from the former ones. For a number of many-objective test problems, all proposed substitute distance assignments resulted into a strongly improved performance of the NSGA-II.

1 Introduction

Recently, there has been increasing awareness for the specific application of evolutionary multi-objective optimization algorithms to problems with a number of objectives considerably larger than two or three. Fleming et al. [8] note the common appearance of such problems in design optimization, and suggested the use of the term *many-objective optimization*. Most evolutionary multi-objective optimization algorithms (EMOs) show a rather decreasing performance, or rapidly increasing search effort for an increasing number of objectives. Other problems with the handling of many objectives are related to the missing means for performance assessment, to difficulties in visualizing results, and to the low number of existing, well-studied test problems. The DTLZ suite of test problems [6,7] defines most of their problems for an arbitrary number of objectives. Results here have been reported for up to 8 objectives [10]. The Pareto-Box problem [12] was also defined for an arbitrary number of objectives, and results were given for up to 15 objectives.

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The reason for the decreasing algorithm performance is strongly related to the (often even exponentially) growing problem complexity. This growing complexity can be measured by several means. One example for this is, if considering a randomly initialized population, the rapidly decreasing probability of having a pair of solutions, where one solution Pareto-dominates the other. Within the unit hypercube, the expectation value for the number of non-dominated solutions among m randomly selected solutions can be computed by [12]:

$$e_m(n) = m - \sum_{k=1}^{m} \frac{(-1)^{k+1}}{k^{n-1}} \binom{m}{k} \tag{1}$$

where m stands for the number of individuals, and n for the number of objectives. For example, for 15 objectives and 10 individuals, the expectation value for the number of dominated solutions is already as low as 0.0027.

Among the most successful and most often applied EMOs we find the Fast Elitist Non-dominated Sorting Genetic Algorithm (NSGA-II) [3,5]. But the poor performance of the NSGA-II algorithm for a large number of objectives has already been reported as well, see e.g. [10,9]. This can be considered a kind of misfortune, as otherwise, the NSGA-II is one of the most attractive EMOs today, due to its simple structure, its availability, the elaborated design of its operations [1], the existence of experience in practical applications, and its excellent performance on the majority of test problems.

This paper attempts to overcome this drawback by analyzing the reasons for NSGA-II's failure in the many-objective optimization domain, and by providing corresponding countermeasures. The main approach, as will be more detailed in section 2, is to replace the crowding distance assignment that is used for secondary ranking among individuals of the same rank. Four methods will be considered here, which all suit better to a larger number of objectives. Section 3 will present results for the convergence metric and Pareto front coverage for a number of many-objective test problems, and section 4 will render conclusions from these results.

2 Substitute Distance Assignments in NSGA-II

2.1 Structure of NSGA-II Algorithm

The outline of the NSGA-II algorithm can be seen in the following listing. Here, we are focussing on a multi-objective minimization problems.

NSGA-II:

$R_t = P_t \cup Q_t$	combine parent and children population
$F = \texttt{fast\_nondominated\_sort}(R_t)$	$F = (F_1, F_2, \cdots)$
	all non-dominated fronts of R_t
$P_{t+1} = \emptyset$, $i = 1$	init next parent population
while $\lvert P_{t+1} \rvert < N$ do	until the parent population is filled