Pareto-, Aggregation-, and Indicator-Based Methods in Many-Objective Optimization

Tobias Wagner[1], Nicola Beume[2], and Boris Naujoks[2]

[1]Institut für Spanende Fertigung (ISF)
[2]Chair of Algorithm Engineering
University of Dortmund, 44221 Dortmund, Germany
wagner@isf.de,{nicola.beume,boris.naujoks}@uni-dortmund.de
http://ls11-www.cs.uni-dortmund.de, http://www.isf.de

Abstract. Research within the area of Evolutionary Multi-objective Optimization (EMO) focused on two- and three-dimensional objective functions, so far. Most algorithms have been developed for and tested on this limited application area. To broaden the insight in the behavior of EMO algorithms (EMOA) in higher dimensional objective spaces, a comprehensive benchmarking is presented, featuring several state-of-the-art EMOA, as well as an aggregative approach and a restart strategy on established scalable test problems with three to six objectives. It is demonstrated why the performance of well-established EMOA (NSGA-II, SPEA2) rapidly degradates with increasing dimension. Newer EMOA like ε-MOEA, MSOPS, IBEA and SMS-EMOA cope very well with high-dimensional objective spaces. Their specific advantages and drawbacks are illustrated, thus giving valuable hints for practitioners which EMOA to choose depending on the optimization scenario. Additionally, a new method for the generation of weight vectors usable in aggregation methods is presented.

1 Introduction

In the field of evolutionary multi-objective optimization, a lot of test problems and applications with two or three objectives have been studied. Problems with more than three objectives, which have been termed *many*-objective problems by Farina and Amato [1], have been tackled only rarely. Many techniques that work well for only a few objectives are anticipated to have difficulties in high-dimensional objective spaces. Thus, many-objective optimization is significantly more challenging than scenarios usually being analyzed.

Within multi-objective optimization, we consider d-dimensional vectors of objective values for a problem of d objective functions $\mathbf{f} = (f_1, \ldots, f_d)$. Among these vectors, a partial order holds concerning the considered minimization problems. For details on often used terms and definitions like Pareto dominance, Pareto set and front, books on EMOA by Deb [2] or Coello Coello et al. [3] are suggested.

The selection module of an EMO algorithm (EMOA) requires a mapping of an objective vector to a ranking criterion to establish a complete order among

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individuals. Popular EMOA usually consist of two selection operators. The primary selection operator is based on Pareto dominance and favors non-dominated solutions over dominated ones. The secondary operator is constituted diversity preserving and rates solutions incomparable concerning the primary operator.

This concept of selection already documents the insight that Pareto dominance may not be sufficient as a sole selection operator, due to the large amount of possibly incomparable solutions. More precisely, a d-dimensional objective vector is only comparable with a fraction of $1/2^{d-1}$ of an (infinite) objective space (cf. Farina and Amato [1]). The importance of the secondary selection operator grows with increasing dimension of the objective space since the incomparability concerning the Pareto-based operator becomes the typical case.

Few previous studies on many-objective optimization by Purshouse and Fleming [4] and Hughes [5] focus to demonstrate the bad performance of NSGA-II by Deb et al. [6]. Hughes observed a simple single-objective restart strategy outperforming NSGA-II on a six-objective function in a two-dimensional decision space. Upon this, he implied a generalization to all Pareto-based techniques.

In contradiction, the work at hand includes positive results by demonstrating that some modern EMOA using Pareto-concepts cope very well with high-dimensional objective spaces. We ascribe the good performance of ε-MOEA, IBEA, SMS-EMOA, and MSOPS to new concepts of aggregation and indicator functions and explain how and why these EMOA work successfully. A comprehensive benchmark is presented on the established test functions of the DTLZ function family, which feature a high dimensional decision and a scalable objective space. Moreover, a slight modification to NSGA-II is suggested, which causes a better performance. Our motivation is not to modify NSGA-II but to demonstrate which aspects of classic EMOA are responsible for the problems within many-objective optimization.

The aggregation method MSOPS by Hughes [5] is studied more detailedly. The problems using aggregation are described and solution concepts are presented with a focus on suitable sets of weight vectors.

The considered test functions, performance measures and basic settings of the EMOA are described in the following section. Section 3 deals with the behavior of Pareto-based EMOA, Section 4 with aggregation methods, and Section 5 with methods utilizing indicator functions for selection. In these sections, algorithms are presented and their performances are described with help of the quality measures. Section 6 summarizes the findings and gives an outlook on how to further deepen insight in many-objective optimization.

2 Benchmark Settings

All algorithms, except otherwise mentioned, have been implemented within the PISA framework[1] [7] since an integrative framework simplifies comparisons. The same variation operators are used with exactly the same parameterization, which

[1] PISA - Platform and Programming Language Independent Interface for Search Algorithms, ETH Zürich (www.tik.ee.ethz.ch/pisa/)