Test Problems Based on Lamé Superspheres

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Abstract. Pareto optimization methods are usually expected to find well-distributed approximations of Pareto fronts with basic geometry, such as smooth, convex and concave surfaces. In this contribution, test-problems are proposed for which the Pareto front is the intersection of a Lamé supersphere with the positive $\mathbb{R}^n$-orthant. Besides scalability in the number of objectives and decision variables, the proposed test problems are also scalable in a characteristic we introduce as *resolvability of conflict*, which is closely related to convexity/concavity, curvature and the position of knee-points of the Pareto fronts.

As a very basic bi-objective problem we propose a generalization of Schaffer's problem. We derive closed-form expressions for the efficient sets and the Pareto fronts, which are arcs of Lamé supercircles. Adopting the bottom-up approach of test problem construction, as used for the DTLZ test-problem suite, we derive test problems of higher dimension that result in Pareto fronts of superspherical geometry.

Geometrical properties of these test-problems, such as concavity and convexity and the position of knee-points are studied. Our focus is on geometrical properties that are useful for performance assessment, such as the dominated hypervolume measure of the Pareto fronts. The use of these test problems is exemplified with a case-study using the SMS-EMOA, for which we study the distribution of solution points on different 3-D Pareto fronts.

1 Introduction

Next to introducing a manageable mathematical foundation for meta-heuristic approaches in multiobjective optimization, constructing a repository of scalable and multimodal test problems is of vital importance [1, 3, 6, 8, 10]. In analyzing a test problem family with well-defined properties, we make a contribution to this ongoing effort of the multiobjective optimization community.

One reason for obtaining the complete Pareto front (PF) of a problem instead of a single non-dominated solution, is that the shape of the PF provides the decision maker with useful extra information about the nature of the conflict. A qualitative approach to this problem is to distinguish between concave, convex and linear (parts of) PFs, as it is well known that on convex PFs it is easier to

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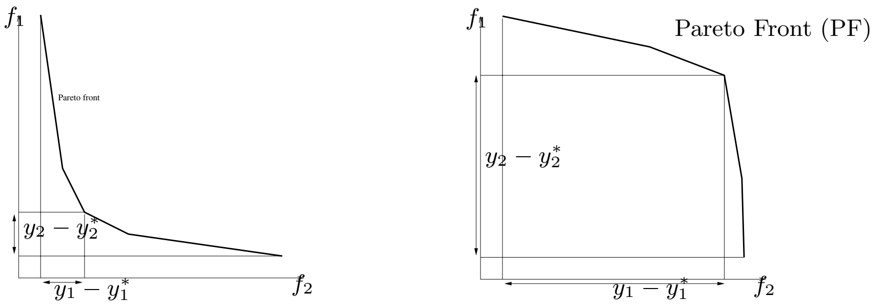

Fig. 1. Visualization of a measure for conflict resolvability. The left figure displays a scenario in which a good compromise exists, while in the scenario displayed on the right hand side they do not exists. The conflict resolvability is computed as the maximum of $y_1 - y_1^*$ and $y_2 - y_2^*$ at the position which minimizes this value, where y_1^* and y_2^* are the coordinates of the ideal point.

find good compromises than on concave ones, and also the behavior of algorithms is often different on both types of geometry.

Taking this into account, we aim for test problems that capture all three types of PFs (concave, convex, and linear). However, the test-case we propose allows also to scale quantitatively the resolvability of conflict. As a measure of the *resolvability of conflict* one may consider:

$$\mathrm{RoC}(PF) = 1 - \frac{\min_{\mathbf{y} \in PF} \max_{i \in \{1,\ldots,m\}} |y_i - y_i^*|}{\max_{\mathbf{y} \in PF} \max_{i \in \{1,\ldots,m\}} |y_i - y_i^*|} \tag{1}$$

Here, $\mathbf{y}^*$ denotes the ideal solution, m is the number of objective functions, and PF denotes the PF. Note, that in case of the denominator being zero, we define $\mathrm{RoC}(PF)$ to be one. Ideally, this value should be close to 1, meaning that all objectives are complementary.

In Figure 1 two situations are depicted. In the left figure a convex PF for a problem is depicted, for which good compromise solutions exists. In the right figure, a concave PF is depicted for which there exists no good *compromise*. We construct a highly symmetrical class of functions for which the geometry of the PF can be varied gradually from convex shapes with high resolvability of conflicts, to linear shapes, and concave shapes with low resolvability of conflicts (cf. figure 1). The problem family we propose is highly symmetrical and only introduces the difficulty of obtaining well-spread solutions on the different shapes of PFs. We consider these problems as interesting, as they can be used for analyzing metaheuristics in a controlled way, i. e. by isolating difficulties. However, we note that the complexity of the test problems can be gradually increased by adding difficulties in a managed way.

The problems can be considered as generalizations of models with spherical symmetries, that are frequently used as elementary test problems in single