Reliability-Based Multi-objective Optimization Using Evolutionary Algorithms

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Abstract. Uncertainties in design variables and problem parameters are inevitable and must be considered in an optimization task including multi-objective optimization, if reliable optimal solutions are to be found. Sampling techniques become computationally expensive if a large reliability is desired. In this paper, first we present a brief review of statistical reliability-based optimization procedures. Thereafter, for the first time, we extend and apply multi-objective evolutionary algorithms for solving two different reliability-based optimization problems for which evolutionary approaches have a clear niche in finding a set of reliable, instead of optimal, solutions. The use of an additional objective of maximizing the reliability index in a multi-objective evolutionary optimization procedure allows a number of trade-off solutions to be found, thereby allowing the designers to find solutions corresponding to different reliability requirements. Next, the concept of single-objective reliability-based optimization is extended to multi-objective optimization of finding a reliable frontier, instead of an optimal frontier. These optimization tasks are illustrated by solving test problems and a well-studied engineering design problem. The results should encourage the use of evolutionary optimization methods to more such reliability-based optimization problems.

1 Introduction

For practical optimization studies, reliability-based techniques are getting increasingly popular, due to their ability to handle uncertainties involved in realizing decision variables and stochasticities involved in various problem parameters. For a canonical deterministic optimization task, the optimum solution usually lies on a constraint surface or at the intersection of more than one constraint surfaces. However, if the design variables or some system parameters cannot be achieved exactly and are uncertain with a known probability distribution of variation, the deterministic optimum (lying on one or more constraint surfaces) will fail to remain feasible in many occasions [5,10]. In such scenarios, a stochastic optimization problem (also known as *chance programming*) is usually formed

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and solved, in which the constraints are converted into probabilistic constraints meaning that probability of failures (of being a feasible solution) is limited to a pre-specified value (say $(1 - R)$) [6], where R is called the reliability of design.

Existing reliability-based optimization techniques vary from each other in the manner they handle the probabilistic constraints. One simple-minded approach would be to use a Monte-Carlo simulation technique to create a number of samples following the uncertainties and stochastitices in the design variables and problem parameters and evaluate them to compute the probability of failure [12,2]. However, such a technique becomes computationally expensive when the desired probability of failure is very small. To alleviate this computational problem, more sophisticated sampling techniques are suggested.

Recently, optimization-based methodologies, instead of sampling methods, are suggested to take care of the probabilistic constraints. In these methods, stochastic variables and parameters are transformed into the standard normal variate space and a separate optimization problem is formulated to compute the largest probability of failure and equate it with the desired value. At least three different concepts – double-loop methods, single-loop methods and decoupled methods – have been followed. In this paper, for the first time, we extend one of these methodologies and apply it with an evolutionary algorithm to solve two different types of optimization problems and demonstrate by solving test problems and an engineering design problem that the evolutionary optimization based reliability consideration is quite appropriate for these problems.

2 Existing Reliability-Based Methodologies

We consider here a reliability-based single-objective optimization problem of the following type:

$$\begin{array}{lll} \underset{(\mathbf{x},\mathbf{d})}{\text{Minimize}} & f(\mathbf{x},\mathbf{d},\mathbf{p}), & \\ \text{Subject to} & g_j(\mathbf{x},\mathbf{d},\mathbf{p}) \geq 0, & j = 1,2,\ldots,J, \\ & h_k(\mathbf{d}) \geq 0, & k = 1,2,\ldots,K, \\ & \mathbf{x}^{(L)} \leq \mathbf{x} \leq \mathbf{x}^{(U)}, \quad \mathbf{d}^{(L)} \leq \mathbf{d} \leq \mathbf{d}^{(U)}. & \end{array} \tag{1}$$

Here, $\mathbf{x}$ is a set of design variables which are uncertain, $\mathbf{d}$ is a set of deterministic design variables, and $\mathbf{p}$ is a set of uncertain parameters (which are not design variables). Thus, the stochasticity in the optimization problem comes from two sets of variables: $\mathbf{x}$ and $\mathbf{p}$. Here, we only consider inequality constraints. This is because if an equality constraint involves $\mathbf{x}$ or $\mathbf{p}$, there may not exist a solution for any arbitrary desired reliability against failure. All inequality constraints can be classified into two categories: (i) stochastic (or chance) constraints g_j involves at least one random variables ($\mathbf{x}$, $\mathbf{p}$ or both) and (ii) h_k involves no random variables. Figure 1 shows a hypothetical problem with two inequality constraints. Typically, the optimal solution lies on a constraint boundary or at the the intersection of more than one constraints, as shown in the figure. In the event of uncertainties in design variables, as shown in the figure with a probability distribution around the optimal solution, in many instances such a solution will be infeasible.