1 Some basics of the single-phase boundary layer theory

Hundreds of very useful constitutive relations describing interactions in multi-phase flows are based on the achievements of the single-phase boundary layer theory. That is why it is important to recall at least some of them, before stepping to more complex interactions in the multi-phase flow theory. My favorite book to begin with learning the main ideas of the single-phase boundary layer theory is the famous monograph by Schlichting (1982). This chapter gives only the basics which helps in understanding the following chapters of this book.

1.1 Flow over plates, velocity profiles, share forces, heat transfer

Consider continuum flow parallel to a plate along the $x$-axis having velocity far from the surface equal to $u_\infty$. The share force acting on the surface per unit flow volume is then

$$f_w = \frac{F_w}{V_{flow}}, \quad (1.1)$$

where the wall share stress is usually expressed as

$$\tau_w = c_w(x) \frac{1}{2} \rho u_w^2. \quad (1.2)$$

Here the friction coefficient $c_w$ is obtained from the solution of the mass and momentum conservation at the surface.

1.1.1 Laminar flow over one site of a plane

For laminar flow over one site of a plane, the solution of the momentum equation delivers the local share stress as a function of the main flow velocity and of the distance from the beginning of the plate as follows
\[ c_w(x) = \frac{\tau_w(x)}{\frac{1}{2} \rho u^2_w} = \frac{0.332}{\left( \frac{u^+_w}{V} \right)^{1/2}}, \]  
(1.3)

Schlichting (1982) Eq. (7.32) p. 140. The averaged drag coefficient over \( \Delta x \) is then

\[ \frac{c_{w,\Delta x}}{\frac{1}{2} \rho u^2_w} = \frac{F_w/(\Delta y/\Delta x)}{\left( \frac{u^+_w}{V} \right)^{1/2}}, \text{ Re}_{\Delta x} < 5 \times 10^5, \]  
(1.4)

Eq. (7.34) Schlichting (1982) p. 141. The corresponding heat transfer coefficients \( h \) are reported to be

\[ Nu_x = \frac{hx}{\lambda} = \frac{1}{\sqrt{\Pr}} \left( \frac{u^+_w}{V} \right)^{1/2} \Pr^{1/2} \text{ for } \Pr \to 0 \text{ for liquid metals}, \]  
(1.5)

\[ Nu_x = \frac{hx}{\lambda} = 0.332 \left( \frac{u^+_w}{V} \right)^{1/2} \Pr^{1/3} \text{ for } 0.6 < \Pr < 10, \]  
(1.6)

\[ Nu_x = \frac{hx}{\lambda} = 0.339 \left( \frac{u^+_w}{V} \right)^{1/2} \Pr^{1/3} \text{ for } \Pr \to \infty, \]  
(1.7)

Schlichting (1982) p. 303. Averaging over \( \Delta x \) results in

\[ \overline{Nu_{\Delta x}} = 2Nu_{\Delta x}. \]  
(1.8)

### 1.1.2 Turbulent flow parallel to plane

For turbulent flow over one site of a plane the solution of the momentum equation gives the local shear stress as a function of the main flow velocity and of the distance from the beginning of the plate as follows

\[ c_w(x) = \frac{\tau_w(x)}{\frac{1}{2} \rho u^2_w} = \frac{0.0296}{\left( \frac{u^+_w}{V} \right)^{1/5}}, \]  
(1.9)

Eq. (21.12) Schlichting (1982) p. 653. This equation is obtained assuming the validity of the so-called 1/7-th velocity profile,