The neutrinoless double-beta decay: A test for new physics

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Abstract. The neutrinoless double-beta decay is not allowed in the Standard Model (SM) but it is allowed in most Grand Unified Theories (GUTs). The neutrino must be a Majorana particle (identical with its antiparticle) and must have a mass to allow the neutrinoless double-beta decay. Apart of one claim that the neutrinoless double-beta decay in $^{76}$Ge is measured, one has only upper limits for this transition probability. But even the upper limits allow to give upper limits for the electron Majorana neutrino mass and upper limits for parameters of GUTs and the minimal $R$-parity violating supersymmetric model. One further can give lower limits for the vector boson mediating mainly the right-handed weak interaction and the heavy mainly right-handed Majorana neutrino in left-right symmetric GUTs. For that, one has to assume that the specific mechanism is the leading one for the neutrinoless double-beta decay and one has to be able to calculate reliably the corresponding nuclear matrix elements. In the present contribution, one discusses the accuracy of the present status of calculating the nuclear matrix elements and the corresponding limits of GUTs and supersymmetric parameters.

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1 Physics beyond the standard model and the double-beta decay

In the standard model the neutrinoless double-beta decay is forbidden but it is allowed in most Grand Unified Theories (GUTs) where the neutrino is a Majorana particle (identical with its antiparticle) and where the neutrinos have a mass.

In GUTs, each beta decay vertex in fig. 1 can occur in eight different ways (see fig. 2).

The hadronic current changing a neutron into a proton can be left or right handed, the vector boson exchanged can be the light one $W_1$ or the heavy orthogonal combination $W_2$ mediating mainly a right-handed weak interaction and two different leptonic currents changing a neutrino into an electron. So the simple vertex for the beta decay can occur in eight different ways.

With two such vertices and the exchange of a light or a heavy mainly right-handed Majorana neutrino one already has 128 different matrix elements describing the neutrinoless double-beta decay [1] (see fig. 3).

At the $R$-parity violating vertex (see fig. 4) of the up- and the down-quarks and the SUSY electron $\tilde{e}$, one has the $R$-parity violating coupling constant $\lambda_{111}$ which is new compared to the standard model [2]. The formation of a pion by a down- and an up-quark produces a long-range

only for Majorana Neutrinos $\nu = \nu^c$

Fig. 1. Neutrinoless double-beta decay of $^{76}$Ge through $^{76}$As to the final nucleus $^{76}$Se. The neutrino must be a Majorana particle that means identical with its antiparticle and must have a mass to allow this decay.

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neutrinoless double-beta decay transition operator. Due to the short-range Brueckner repulsive correlations between two nucleons, this is increasing the transition probability by a factor 10000. The upper limit derived from upper limits of the neutrinoless transition probability for $^{76}\text{Ge}$ is therefore more stringent and one can derive from the neutrinoless transition probability in $^{76}\text{Ge}$ an upper limit for $|\lambda_{111}| < 10^{-4}$.

To calculate the neutrinoless double-beta decay transition probability, we use Fermi’s Golden Rule in second order:

$$T = \sum_k \frac{\langle \hat{H}_W | k \rangle \langle k | \hat{H}_W | i \rangle}{E_i - E_k},$$

$$E_i = E_{0+}(^{76}\text{Ge}),$$

$$E_k = E_k(e^-) + E_k(\nu) + E_{\text{nuclei}}(^{76}\text{As}; k),$$

$$T = M_m(M_\nu + M_\gamma t \delta \phi) + M_{W_R} \left( \frac{M^2}{M^2_R} \right) + M_{SU SY} \lambda_{111}^2 + M_{V_R} \left( \frac{m_p}{M_{V_R}} \right),$$

$$w = \frac{2\pi}{\hbar} |T|^2 p_f \leq 4.4 \cdot 10^{-33} \text{ [s}^{-1}] .$$

Here $|k\rangle$ are the intermediate nuclear states in $^{76}\text{As}$ with a Majorana neutrino and one electron. The sum over $k$ includes also an integration over neutrino energies. $E_{0+}(^{76}\text{Ge})$ is the ground-state energy of $^{76}\text{Ge}$.

To calculate the nuclear matrix elements, the most reliable method has turned out to be the Quasiparticle Random Phase Approximation (QRPA) to calculate in the example of $^{76}\text{Ge}$ the wave function of the initial $^{76}\text{Ge}$, the excited states of the intermediate nucleus $^{76}\text{As}$ and the ground state of the final nucleus $^{76}\text{Se}$ [3].

The QRPA approach describes the intermediate excited nuclear states $|m\rangle$ for example in $^{76}\text{As}$ as a coherent superposition of two quasiparticle excitations and two quasiparticle annihilations relative to the initial ground state (in our example) of $^{76}\text{Ge}$:

$$a_1^+ \gamma_1 = \sum_k \left[ a_1^+ \gamma_1^{(k)} \right],$$

$$A_{\nu} = \left[ a_1^+ \gamma_1^{(k)} \right],$$

$$Q^+ = \sum_{\alpha} \left[ X^{m}_{\alpha} A^{+}_{\alpha} - Y^{m}_{\alpha} A_{\alpha} \right],$$

$$|m\rangle = Q^+ |g\rangle .$$

(2)

The QRPA equation for the determination of the intermediate states by $X^{m}_{\alpha}$ and $Y^{m}_{\alpha}$ is derived from the many-body Schroedinger equation by using quasiboson commutation relations for the quasiparticle pairs $A_1^+$, One uses therefore for deriving the QRPA equations that two quasiparticle states behave like bosons. This is a usual approximation one often is using in physics. For example, one describes a pair of quarks and antiquarks as a meson and treats it as a boson, although it consists out of a fermion pair.

To test the quality of this approximation for the two neutrino and the neutrinoless double-beta decay, we include the exact commutation relations of the fermion pairs at least as ground-state expectation values. This is called renormalized-QRPA (R-QRPA) applied in [4] to the two neutrino double-beta decay and in [5] for the first time to the interesting neutrinoless double-beta decay.

The R-QRPA includes the Pauli principle into the QRPA and reduces by that the number of quasiparticles.