Testing Stochastic Arithmetic and CESTAC Method Via Polynomial Computation

René Alt[1], Jean-Luc Lamotte[1], and Svetoslav Markov[2]

[1] CNRS, UMR 7606, LIP6,University Pierre et Marie Curie, 4 place Jussieu, 75252 Paris cedex 05, France
Rene.Alt@lip6.fr, Jean-Luc.Lamotte@lip6.fr
[2] Institute of Mathematics and Informatics, Bulgarian Academy of Sciences, "Acad. G. Bonchev" st., bl. 8, 1113 Sofia, Bulgaria
smarkov@bio.bas.bg

Abstract. The CESTAC method and its implementation known as CADNA software have been created to estimate the accuracy of the solution of real life problems when these solutions are obtained from numerical methods implemented on a computer. The method takes into account uncertainties on data and round-off errors. On another hand a theoretical model for this method in which operands are gaussian variables called stochastic numbers has been developed. In this paper numerical examples based on the Lagrange polynomial interpolation and polynomial computation have been constructed in order to demonstrate the consistency between the CESTAC method and the theory of stochastic numbers. Comparisons with the interval approach are visualized.

1 Introduction

The CESTAC method is an approach to deal with numerical problems involving uncertainties. It has been created to estimate the accuracy of the solution of real life problems when these solutions are obtained from numerical methods implemented on a computer. Such applications to real life problems can be found in [4], [6], [10] and [11]. This method is of Monte-Carlo-type and consists in performing each arithmetic operation several times using an arithmetic with a random rounding mode, see [2], [12], [13]. In other words, real numbers are considered as random values with some prescribed probabilities. In the simplest case one considers gaussian distributed random values, so-called *stochastic numbers*. Stochastic numbers possess only two probability parameters: mean value and standard deviation, and allow for simple arithmetic operations over them. Working with them can be considered as a particular case of granular computing in the same way as it has been done for intervals [9]. The difference is that here, intervals are confidence intervals and the operations on them are also different. The classical operations on gaussian continuous functions is called *Stochastic Arithmetic* or more precisely *Continuous Stochastic Arithmetic* (*CSA*).

In the CESTAC method a stochastic number is represented by several, say k, samples $x_j,\ j = 1, ..., k$, representing a given number x. The operations on these

P. Melin et al. (Eds.): IFSA 2007, LNAI 4529, pp. 13–22, 2007.

samples are those of the computer in use followed by a random rounding. The samples are randomly generated in a known confidence interval. The mean value $\overline{x}$ is the best approximation of the exact value x and the number of significant digits on $\overline{x}$ is computed by:

$$C_{\overline{x}} = log_{10}\left(\frac{\sqrt{k}\,|\overline{x}|}{\sigma\ \tau_\eta}\right), \tag{1}$$

wherein

$$\overline{x} = \frac{1}{k}\ \sum_{j=1}^{k} x_j, \qquad \sigma^2 = \frac{1}{k-1}\ \sum_{j=1}^{k}(x_j - \overline{x})^2$$

and τ_η is the value of the Student distribution for $k-1$ degrees of freedom and a probability level 0.95. This type of computation on samples approximating the same value is called *Discrete Stochastic Arithmetic* (*DSA*).

Operations on stochastic numbers are used as a model for operations on imprecise numbers, i. e. real numbers containing an unknown error, which is supposed to be centered gaussian with a known standard deviation. Some fundamental properties of stochastic numbers are considered in [3], [14].

This work is part of a more general one, which consists in studying the algebraic structures induced by the operations on stochastic numbers in order to provide a good algebraic understanding of the performance of the CESTAC method [1], [7], [8].

The operations addition and multiplication by scalars are well-defined for stochastic numbers and their properties have been studied in some detail. More specifically, it has been shown that the set of stochastic numbers is a commutative monoid with cancelation law in relation to addition. The operator multiplication by -1 (negation) is an automorphism and involution. These properties imply a number of interesting consequences, see, e. g. [7], [8].

In the sequel we first briefly present some algebraic properties of the system of stochastic numbers with respect to the arithmetic operations addition, negation, multiplication by scalars, multiplication between two stochastic numbers and the relation inclusion. This theoretical results are the bases for the numerical experiments presented in the second part of the paper.

2 Stochastic Arithmetic Theory (SAT) Approach

A *stochastic number* a is written in the form $a = (a'; a'')$. The first component a' is interpreted as *mean value*, and the second component a'' is the *standard deviation*. A stochastic number of the form $(a'; 0)$ has zero standard deviation and represents a (pure) mean value, whereas a stochastic number of the form $(0; a'')$ has zero mean value and represents a (pure) standard deviation. In this work we shall always assume $a'' \geq 0$; however, in some cases it is convenient to consider negative standard deviations. Denote by $\mathbb{S}$ the set of all stochastic numbers, $\mathbb{S} = \{(a'; a'') \mid a' \in \mathbb{R}, a'' \in \mathbb{R}^+\}$.