Friction Model by Using Fuzzy Differential Equations

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Abstract. In the present paper we propose a novel approach for modeling friction, by using fuzzy differential equations under the strongly generalized differentiability concept. The key point is a continuous fuzzyfication of the signum function. The lack of the uniqueness for the solutions of a fuzzy differential equation allows us to choose the solution which better reflects the behavior of the modeled real-world system, so it allows us to incorporate expert's knowledge in our model. Numerical solutions of the fuzzy differential equations modeling dry friction are proposed. In order to show how the expert's knowledge can be incorporated in the system, we study the dry friction equation with different additional assumptions.

1 Introduction

The existing models of the friction forces show discontinuous variation at the zero transition of the velocity (see e.g. [5], [19], [17], [11], [12]). The effects of friction at low velocities are due to local properties of the materials and an accurate model of these phenomena is possible by taking into account properties both at the molecular and macroscopic level ([19], [16]). Since the information on the molecular level is usually unavailable we have uncertianties. These uncertainties are usually modeled by considering the friction force as a multivalued function and in this case the equations of motion are considered as differential inclusions. This approach is used in several works (see [1], [15], [8], etc.). The idea behind using differential inclusions is substituting the signum function by a multivalued function. This model often manifests chaotic behavior ([11], [10]).

The above discussion shows that the model of a system with friction is often subject of non-statistical uncertainties. So, in order to model the behavior of a system under the presence of the friction forces we have to take into account these uncertainties. In order to take into account these uncertainties we propose

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in the present paper an alternative fuzzy model based on fuzzy differential equations (FDEs). Surely other techniques can be easily imagined (such as interval methods) but these are subject of further research. Also, in our proposed method it is possible to incorporate expert knowledge about the system under study and this property can be turned into an advantage in future studies.

FDEs appear naturally as tools for modeling dynamical systems under uncertainty. Till now, they are rarely used in modeling real-world systems since their theory was developed relatively recently. Also, as it is shown in several recent papers, FDEs are not just an easy extension of the theory of ODEs to the fuzzy case (see e.g. [13], [14], [2]). This fact is also slowing down the extension of the applicability of FDEs. There are several different interpretations of the notion of a FDE (for a discussion about them please refer to [2]). In the present paper we will use the so called strongly generalized differentiability concept introduced recently as a method which solved some problems with the other FDE interpretations (H-derivative (see [18]) or fuzzy differential inclusions (see [7])).

Strongly generalized differentiability was introduced in [3]. The strongly generalized derivative is defined for a larger class of fuzzy-number-valued functions than the H-derivative and fuzzy differential equations can have solutions with decreasing length of their support (this was not the case for the H-derivatve). Also, contrary to the case of differential inclusions, the derivative concept for fuzzy-number-valued function is defined and this makes this method more appropriate for numerical computations. First order linear fuzzy differential equations are investigated in [4] and the behavior of their solutions motivate also the use of the above cited results in the present paper for building a novel friction model.

The key point in our discussion is how to fuzzify the classical model in order to get meaningful conclusions. The key role in this fuzzification is played by the frictional term in the equations of movement with friction. In the present paper, following [7], we fuzzify in a heuristic way the Signum function, by making this term also continuous fuzzy valued function. However we have gained the continuity of the frictional term, since it is a fuzzy one, we obtain a fuzzy solution for our model. The interpretation of this model is the fuzzy set of trajectories, attainable by the system ([7]). The lack of uniqueness of the solution of a fuzzy differential equation under the generalized differentiability concept at first sight could be seen as a disadvantage. But it is turned into an advantage in the present paper since we are able this way to include in our model knowledge based on observations of the modeled system. In the present paper we do not deal with the problem of control, this being subject of further research.

After a preliminary section we propose in Section 3 the heuristic fuzzy model for friction forces with a discussion on dry friction. In Section 4 we present also some preliminary results how friction is modeled by using the proposed approach. We end up with some conclusions and further research topics.