Non-commutative System of Fuzzy Interval Logic Generated by the Checklist Paradigm Measure m_3 Containing Early Zadeh Implication

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Abstract. This paper continues investigation of systems of fuzzy interval logics based on the Checklist Paradigm semantics of Bandler and Kohout [1] [2]. While the early papers dealt with checklist paradigm based interval systems containing commutative AND and OR, this paper is the fifth in the series of papers in which we have been describing the systems in which these connective types are non-commutative. In the present paper we investigate non-commutative interval system generated from implication operators based on the Checklist Paradigm measure m_3 of Bandler and Kohout. This system includes the well-known Early Zadeh implication operator (PLY) which is not contrapositive. While the commutative systems can be sufficiently characterized by an 8-element group of transformations, the non-commutative systems require the 16 element group $\mathcal{S}_{2\times2\times2\times2}$.

1 Introduction

The major theme of this paper is a non-commutative interval system generated from implication operators based on the measure m_3 of Bandler and Kohout by the group transformations provided by the group $\mathcal{S}_{2\times2\times2\times2}$. This system includes the well-known Early Zadeh implication operator (PLY) which is not contrapositive. It generates system of connectives which forms a subgroup $\mathcal{S}_{2\times2\times2}$ of the 16 element group $\mathcal{S}_{2\times2\times2\times2}$. Non-contrapositivity of PLY induces non-commutativity of AND and OR connectives under the group transformations. Section 2.2 deals with group transformations of this system, while section 4 describes the system and also overviews three different classifications of connectives of this interval system.

1.1 Interval Logics Generated by the Checklist Paradigm

In 1979 Bandler and Kohout [3] derived five interesting systems from *ab initio* principles based on the Checklist paradigm. The structure of each of these fuzzy

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interval systems is generated by a distinct measure that performs the *summarization* of the information contained in certain well-defined binary structures called *fine structures*. See the Appendix below for the definitions of the measures we refer to in this section. The interval produced by a measure m_i pair of connectives of one type can be generically characterized by the following inequality:

$$conbot \leq m_i \leq contop$$

For example, Bandler and Kohout [3] listed the following five inequalities linking the interval bounds for implication operators $\rightarrow_{bot}, \rightarrow_{top}$ with corresponding measures[1] $m_i,\ i = \{1, 2, 3, 4, 5\}$:

1. The Kleene-Dienes implication(KD) and Łukasiewicz implication (Ł) respectively, are attainable lower and upper bounds of m_1:
   $min(1, 1 - a + b) \geq m_1(\rightarrow) \geq max(1 - a, b)$
2. A certain new function of (a, b) and the Goguen-Gaines (G43) implication (the left-hand side) are respectively attainable lower and upper bounds of m_2:
   $min(1, b/a) \geq m_2(\rightarrow) \geq max(0, (a + b - 1)/a)$,
3. Another function of (a, b) and the Early Zadeh implication (EZ) are respectively attainable lower and upper bounds of m_3:
   $max[min(a, b), 1 - a] \geq m_3(\rightarrow) \geq max(a + b - 1, 1 - a)$.
4. Still another function of (a, b) and the Wilmott implication (W) respectively, are attainable lower and upper bounds of m_4:
   $min[max(a + b - 1, 1 - a), max(b, 1 - a - b)] \leq m_4(\rightarrow) \leq min[max(1 - a, b), (max(a, (1 - b), min(b, 1 - a)))]$
5. Yet another function of (a, b) and one of G43 respectively, are attainable lower and upper bounds of m_5:
   $max[min(1, b/a), 1 - a] \geq m_5(\rightarrow) \geq max[(a + b - 1)/a, 1 - a]$.

The above quoted paper [3] gave the impetus for more systematic investigation of the systems of connectives that can be generated for above listed implicational intervals by group transformations. The formal semantics for all the interval systems so far described in various papers by Bandler and Kohout, Kohout and Bandler and also by Kohout and Kim are derived by means of an exact mathematical method, which also has a sound ontological and epistemological base. It is based on the **checklist paradigm** introduced by Bandler and Kohout [3],[4],[5],[6]. In order to make this paper self-explanatory, a brief overview of the Checklist Paradigm is provided in the Appendix. The definitions of measures $m_i,\ i = \{1, 2, 3, 4, 5\}$ is also presented in the Appendix.

2 The Structure of Some Classes of Fuzzy Interval Logic Systems

The *checklist paradigm* puts ordering on the *pairs of* distinct implication operators and other pairs of connectives. Hence, it provides a theoretical justification of *interval-valued* approximate inference.

[1] See the Appendix below for the definitions of the measures we refer to in this section.