Atanassov's Intuitionistic Fuzzy Sets as a Classification Model

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Abstract. In this paper we show that Atanassov's Intuitionistic Fuzzy sets can be viewed as a classification model, that can be generalized in order to take into account more classes than the three classes considered by Atanassov's (membership, non-membership and non-determinacy). This approach will imply, on one hand, to change the meaning of these classes, so each one will have a positive definition. On the other hand, this approach implies the possibility of a direct generalization for alternative logics and additional valuation states, being consistent with Atanassov's focuss. From this approach we shall stress the absence of any structure within those three valuation states in Atanassov's model. In particular, we consider this is the main cause of the dispute about Atanassov's model: acknowledging that the name *intuitionistic* is not appropriate, once we consider that a crisp direct graph is defined in the valuation space, formal differences with other three-state models will appear.

Keywords: Atanassov's Intuitionistic Fuzzy Sets, Interval Valued Fuzzy Sets, Type-2 Fuzzy Sets, L-Fuzzy sets.

1 Introduction

The fuzzy scientific community has been attending with great interest the recent dispute about Atanassov's *Intuitionistic* Fuzzy Sets [6,14]. This model was originally proposed by Atannassov [4] as a generalization of Zadeh's Fuzzy Sets [27].

According to Atanassov [4,5], given a set of objects X, each object $x \in X$ is being described by means of the degree of membership, $\mu(x) \in [0,1]$, together with the degree of non-membership, $\nu(x) \in [0,1]$, not imposing as Zadeh that these two values should sum up to 1 (i.e., Atanassov does not assumes that the degree of non-membership is the standard negation of the degree of membership, $\nu(x) = 1-\mu(x), \forall x \in X$). On the contrary, a remaining degree on non-determinacy (*hesitation* margin) is allowed by Atanassov, $\pi(x) = 1 - \mu(x) - \nu(x) \in [0,1], \forall x \in X$.

Atanassov's model has deserved a serious consideration from theoretical and applied researchers: as pointed out in [7], his papers have more than 1000 references in scientific papers, see also [5]. Nevertheless, we should acknowledge

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that the term *intuitionistic* is not appropriate in Atanassov's model (see [9]). A proper *intuitionistic* model was proposed in [23].

Moreover, Atanassov's proposal is not fully clear about the meaning and estimation of the degrees of non-membership and non-determinacy (see, e.g., [14]).

In addition, Atanassov's model, see [13,17], is equivalent to *interval valued fuzzy sets* [22,24], which are defined by the family of mappings

$$\mu : \mathcal{X} \longrightarrow Int([0,1])$$

where $Int([0,1])$ is the set of all closed subintervals in $[0,1]$ and $\mu(x)$ represents the plausible range of the degree to which object $x \in \mathcal{X}$ verifies a certain fuzzy property.

This paper is organized as follows: in section 2 we propose to consider Atanassov's model as a classification model with two main valuation states, plus non-determinacy. In section 3 we propose a direct generalization of his model, allowing more than two main valuation states and providing such a valuation space with a directed graph. In section 4 we stress the consequences of the non-existence of such a graph in Atanassov's model, which we consider the main cause of the confusion with other models. Finally, we conclude with some additional comments about other Atanassov's assumptions that help to explain why such a demanded structure does not seem relevant under his approach.

2 Partition-Based Classification Models

Following [2], we consider here a finite valuation space $\mathcal{C}$ of *valuation states*, fuzzy in nature but well defined from a representation point of view, in such a way that for each object $x \in X$ we can evaluate the degree $\mu_x(c) \in [0,1]$ to which such an object verifies properties defining each class $c \in \mathcal{C}$. No restriction is imposed by definition on these degrees of membership, but the possibility of modifying those values through learning (see also [1]).

In order to be properly defined, each one of those valuation spaces (perhaps represented by a linguistic label), should be positively defined. From this point of view, Atanassov's *non-membership* should be changed into a dual or opposite class, different than negation. For example, the opposite of *tall* is *short* and the opposite of *good* is *bad*. Neither negation of *tall* is *short* or the negation of *good* is *bad*.

Let us remind here that the concept of fuzzy partition, introduced by Ruspini [21] in order to generalize the classical crisp partition concept, assumes the existence of a discrete family $\mathcal{C}$ of *classes*, in such a way that

$$\sum_{c \in \mathcal{C}} \mu_c(x) = 1, \forall x \in X$$

holds. Each object $x \in X$ may belong to several classes -to certain degrees-, and the total degree of membership is distributed among all classes (a crisp partition will appear whenever $\mu_c(x) \in \{0,1\}, \forall c \in \mathcal{C}, \forall x \in X$).