Intuitionistic Fuzzy Histograms of an Image

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Abstract. This paper proposes an automated approach for constructing the intuitionistic fuzzy histograms (IF-histograms) of a gray-scale digital image, based on the notion of intuitionistic fuzzy numbers (IF-numbers). A method for constructing parametric IF-numbers from their fuzzy counterparts using intuitionistic fuzzy generators (IFGs) is also presented, using an entropic optimization criterion. Finally, experimental results demonstrate the ability of the proposed approach to obtain efficiently the IF-histograms of gray-scale images.

1 Introduction

Since Zadeh introduced fuzzy sets (FSs) theory [1], many theories treating imprecision have been proposed. Among the various extensions of FSs, Atanassov's intuitionistic fuzzy sets (A–IFSs) [2,3,4] provide a flexible and intuitive framework to deal with vagueness originating out of imperfect or/and imprecise information. The sound advantage of A–IFSs is that they are consistent with the human behavior of decision making, expressing the fact that linguistic negation does not always coincides with logical negation.

Digital image processing algorithms can be roughly classified into two main categories; histogram- and pixel-based approaches. Many of the algorithms consider and depend solely on the histogram of the image. Histogram-based techniques are characterized by their simplicity and speed compared to their pixel-based counterparts. Therefore, the concept of histogram, as a descriptor of the underlying statistics of images, is very important in the context of image processing.

In this paper, a novel method for constructing the intuitionistic fuzzy histograms (IF-histograms) of digital images is presented. The method is based on the concept of intuitionistic fuzzy numbers (IF-numbers) and intuitionistic fuzzy generators (IFGs).

The paper is organized as follows. In Sect. 2 the basic elements of A–IFSs theory are outlined and the concept of intuitionistic fuzzy generators is briefly discussed. Sect. 3 presents the concepts of fuzzy numbers (F-numbers) and fuzzy histogram (F-histogram), as well as their intuitionistic fuzzy equivalents. An

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entropic optimization approach for selecting the optimal IF-number to construct the IF-histogram is also described. Finally, experimental results are given in Sect. 4, while conclusions are drawn in Sect. 5.

2 Atanassov's Intuitionistic Fuzzy Sets

In this section, we briefly describe the basic notions, concepts, and definitions of A–IFSs theory.

Definition 1. *An FS $\tilde{A}$ defined on a universe X may be given as [1]*

$$\tilde{A} = \{\langle x, \mu_{\tilde{A}}(x)\rangle | x \in X\} \ , \tag{1}$$

where $\mu_{\tilde{A}}(x) : X \to [0,1]$ is the membership function of $\tilde{A}$.

The membership function of $\tilde{A}$ describes the *degree of belongingness* of $x \in X$ in $\tilde{A}$.

Definition 2. *An A–IFS A defined on a universe X is given by [2,3,4]*

$$A = \{\langle x, \mu_A(x), \nu_A(x)\rangle | x \in X\} \ , \tag{2}$$

where

$$\mu_A(x) : X \to [0,1] \qquad \textit{and} \qquad \nu_A(x) : X \to [0,1] \ ,$$

with the condition

$$0 \leqslant \mu_A(x) + \nu_A(x) \leqslant 1 \ , \tag{3}$$

for all $x \in X$.

The values of $\mu_A(x)$ and $\nu_A(x)$ denote the *degree of belongingness* and the *degree of non-belongingness* of x to A, respectively. For an A–IFS A in X we call the *intuitionistic index* of an element $x \in X$ in A the following expression

$$\pi_A(x) = 1 - \mu_A(x) - \nu_A(x) \ . \tag{4}$$

We can consider $\pi_A(x)$ as a *hesitancy degree* of x to A [2,3,4]. From (4) it is evident that

$$0 \leqslant \pi_A(x) \leqslant 1 \tag{5}$$

for all $x \in X$.

FSs can also be represented using the notation of A–IFSs. An FS $\tilde{A}$ defined on X can be represented as the following A–IFS

$$A = \{\langle x, \mu_A(x), 1 - \mu_A(x)\rangle | x \in X\} \ , \tag{6}$$

with $\pi_A(x) = 0$ for all $x \in X$.

Definition 3. *The complementary set A^c of A is defined as*

$$A^c = \{\langle x, \nu_A(x), \mu_A(x)\rangle | x \in X\} \ . \tag{7}$$

Finally, throughout this paper by $\mathscr{IFS}(X)$ we denote the set of all A–IFSs defined on X. Correspondingly, $\mathscr{FS}(X)$ is the set of all FSs on X.