Well-Distributed Pareto Front by Using the $\epsilon \rightarrow \textit{MOGA}$ Evolutionary Algorithm*

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Abstract. In the field of multiobjective optimization, important efforts have been made in recent years to generate global Pareto fronts uniformly distributed. A new multiobjective evolutionary algorithm, called $\epsilon \rightarrow \textit{MOGA}$, has been designed to converge towards $\Theta^\ast_P$, a reduced but well distributed representation of the Pareto set $\Theta_P$. The algorithm achieves good convergence and distribution of the Pareto front $J(\Theta_P)$ with bounded memory requirements which are established with one of its parameters. Finally, a optimization problem of a three-bar truss is presented to illustrate the algorithm performance.

1 Introduction

Many engineering design problems can be translated into multiobjective optimization (MO) problems. MO techniques present advantages over single objective optimization techniques due to the possibility of providing a solution with different trade-offs among different individual objectives of the problem. In that case, the Decision Maker (DM) can select the best final solution according to its preferences.

MO methods provide the designer with the possibility of a better selection of the final solution, since there is no part of the searching space ignored. Solutions provided by MO algorithms should be representative of the whole space of design variables. Since computational algorithms perform a discrete search in the space of design variables, the solutions found must be evenly distributed to avoid over-explored or under-explored areas. On the other hand, that set of solutions should not contain non-optimal ones, since this situation could make the DM selects a inappropriate value for the design variables.

The solution of an MO problem often leads to a family of Pareto optimal points, where any improvement in one objective result in the degradation of one or more of the other objectives. These points, represented in the objective function space, conform the so called Pareto front. Thus, any point is better than another in this front (non-dominated points).

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In [8], MO algorithms based on numerical optimization and random search are analyzed and a new numerical optimization method is proposed, the Normalized Normal Constraint (NNC). It generates well-distributed Pareto fronts, however, the solution obtained is highly dependent on the initial optimization conditions since it uses a search-based Gauss-Newton method. A modified variant of the NNC (MNNC) is presented in [7], which uses a Genetic Algorithm (GA) to obtain global optimum solutions. The MNNC method presents a high computational burden since an independent optimization is needed for each point belonging to the front.

Other alternatives to solve MO problems are by using Multiobjective Evolutionary Algorithms (MOEAs). This kind of algorithms produces simultaneously - in parallel and in a single run - several elements of the Pareto front, thanks to their populational nature. The good results obtained with MOEAs and their capacity to handle a wide variety of problems explain why they are currently one of the areas where most progress is being made within the EAs field [93].

In this paper, a new MOEA algorithm called \(\varepsilon-MOGA\) has been designed to achieve a reduced but well distributed representation of the Pareto front. The algorithm adjusts the limits of the Pareto front dynamically and prevents the solutions belonging to the ends of the front from being lost.

The paper is organized as follows. Section 2 presents the \(\varepsilon\)-MOGA algorithm. Section 3 illustrates the \(\varepsilon\)-MOGA performance with a three-bar truss design example. Finally, some concluding remarks are reported in section 4.

### 2 \(\varepsilon\)-MOGA

The MO problem can be formulated as follow:

\[
\min_{\theta \in D \subseteq \mathbb{R}^L} J(\theta) = \min_{\theta \in D \subseteq \mathbb{R}^L} [J_1(\theta), J_2(\theta), \ldots, J_s(\theta)]
\]

(1)

where \(J_i(\theta), i \in B := [1 \ldots s]\) are the objectives to optimize and \(\theta\) is a solution inside the \(L\)-dimensional solution space \(D\).

To solve the MO problem the Pareto optimal set \(\Theta_P\) (solutions where no-one dominates others) must be found. Pareto dominance is defined as follows.

A solution \(\theta^1\) dominates another solution \(\theta^2\), denoted by \(\theta^1 \prec \theta^2\), iff

\[
\forall i \in B, J_i(\theta^1) \leq J_i(\theta^2) \land \exists k \in B : J_k(\theta^1) < J_k(\theta^2) .
\]

Therefore the Pareto optimal set \(\Theta_P\), is given by

\[
\Theta_P = \{ \theta \in D \mid \nexists \tilde{\theta} \in D : \tilde{\theta} \prec \theta \} .
\]

(2)

\(\Theta_P\) is unique and normally includes infinite solutions. Hence a set \(\Theta_P^*\) (which is not unique), with a finite number of elements from \(\Theta_P\), should be obtained.

The \(\varepsilon\)-MOGA variable (\(\varepsilon\)-MOGA) [4] is an elitist multiobjective evolutionary algorithm based on the concept of \(\varepsilon\)-dominance [6]. \(\varepsilon\)-MOGA obtains an