Practical Stabilization of Nonholonomic Mobile Robots Moving on Uneven Surface

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Abstract. The stabilization problem of nonholonomic mobile robots that move on uneven surface is investigated. Smooth control laws ensuring practical stabilization are proposed based on the transverse function approach and the integrator backstepping technique. Firstly, a transverse function which defines a smooth embedded submanifold is constructed, and an augmentation procedure is applied to the kinematic model of the robots. Then, the left-invariance property of the kinematic model is explored, and after the defining of the error variables via group operation, smooth exponential stabilizing control laws are derived for the error dynamic system, thus the kinematic model is rendered practically stable. Finally, the obtained control laws are utilized in the integrator backstepping process to stabilize the dynamic model. Simulations are provided to demonstrate the effectiveness of the algorithm.

1 Introduction

There have been numerous studies on the control problem of wheeled mobile robots (WMRs) over the past twenty years [1]. One reason is that WMRs are more and more widely used in different settings ranging from shopping centers, hospitals, manufactories for applications such as security, transportation, inspection, planetary exploration, etc. The other reason comes from theoretic challenge. As pointed out by the celebrated work of Brockett [2], nonholonomic systems, which include WMRs as special cases, cannot be asymptotically stabilized by any smooth time-invariant state feedback law. This makes the feedback stabilization problem of nonholonomic systems nontrivial. To deal with the technical obstacle, researchers have proposed three types of controllers that utilize smooth time-varying control laws, discontinuous time-invariant control laws, or a hybrid form of them to solve the stabilization problem. Another basic control problem is trajectory tracking, on which rapidly expanding researches have been carried out in the recent years. For an in-depth overview of the control of nonholonomic systems we refer to the early survey paper [1] and the recent one [3], as well as the references cited therein.

It seems that the problem of controlling certain classes of nonholonomic systems is virtually solved. Nevertheless, there are still many issues that deserve further attention for the WMRs control designer. An interesting issue which has often been overlooked is the control of WMRs moving on an uneven surface. It is assumed in most of the existing literatures that WMRs move on a plane. This is not the case when WMRs are utilized to perform tasks in some outdoor environment, e.g., planetary exploration.
From an engineering perspective, one should take into account the practical ground information to improve the performance of the control laws. An early result in this direction is [4], in which a discontinuous stabilizer is obtained under the high-order generalized chained systems framework.

From the above discussion, the objective of this paper is to present a method for solving the control problem of WMRs moving on an uneven surface. Smooth control laws are proposed based on the combination of transverse function method which is a new and still maturing control approach for nonlinear system [5-9], and the popular backstepping technique.

2 Modeling and Problem Formulation

Consider a differentially driven WMR moving on an uneven surface described by the equation $z = W(x, y)$. Assume that the wheels are rolling on the surface without slipping. It is the case if the friction in the contact point is sufficient large, and the grads of the surface are bounded. For the sake of simplicity, assume further that $W(x, y)$ is a quadratic polynomial described as follows:

$$W(x, y) = W_0 + \frac{1}{2}W_1x^2 + W_2x + W_3xy + W_4y + \frac{1}{2}W_5y^2,$$

where $W_i$, $i = 1, \ldots, 5$ are some known constants. This simplified assumption meets with most real-world situations, at least locally, because any curved surface can be locally approximated by a quadratic surface.

The motion of the mobile robot will be certainly affected by the gravity. A simplified dynamic model of the mobile robot is described by the following equations

$$\begin{align*}
\dot{x} &= v \cos \theta \\
\dot{y} &= v \sin \theta \\
\dot{\theta} &= \omega \\
m\dot{v} &= \frac{\tau_1 + \tau_2}{r} - mg \left( \frac{\partial W}{\partial x} \cos \theta + \frac{\partial W}{\partial y} \sin \theta \right) \\
J\dot{\omega} &= \frac{(\tau_1 - \tau_2)}{r} l
\end{align*}$$

with $\theta$ the heading angle with respect to the $x$-axis of the inertial reference frame, $v$ the translational velocity of the center of mass, and $\omega$ the angular velocity around the vertical axis of the body frame. The radii of the wheels is denoted as $r$, and the distance between the two rare wheels is $2l$. $\tau_1$ and $\tau_2$ are the applied torques of two motors driving the two rear wheels respectively. In what follows, the mass $m$, the moment of inertia $J$, the radii $r$, the distance $l$, and the gravitational constant $g$ are all set to 1 for simplicity. The configuration of the mobile robot is described by the vector $(x, y, z, \theta)^T$, with $(x, y, z)^T$ the coordinates of the center of mass in the inertial