Symmetries in Natural Language Syntax and Semantics: The Lambek-Grishin Calculus

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Abstract. In this paper, we explore the Lambek-Grishin calculus LG: a symmetric version of categorial grammar based on the generalizations of Lambek calculus studied in Grishin [1]. The vocabulary of LG complements the Lambek product and its left and right residuals with a dual family of type-forming operations: coproduct, left and right difference. The two families interact by means of structure-preserving distributivity principles. We present an axiomatization of LG in the style of Curry’s combinatory logic and establish its decidability. We discuss Kripke models and Curry-Howard interpretation for LG and characterize its notion of type similarity in comparison with the other categorial systems. From the linguistic point of view, we show that LG naturally accommodates non-local semantic construal and displacement — phenomena that are problematic for the original Lambek calculi.

1 Background

The basic Lambek calculus [2] is a logic without any structural rules: grammatical material cannot be duplicated or erased without affecting well-formedness (absence of Contraction and Weakening); moreover, structural rules affecting word order and constituent structure (Commutativity and Associativity) are unavailable. What remains (in addition to the preorder axioms for derivability) is the pure logic of residuation of (1).

\[
\text{reshuated triple} \quad A \rightarrow C/B \text{ iff } A \otimes B \rightarrow C \text{ iff } B \rightarrow A \setminus C \quad (1)
\]

The type-forming operations have two kinds of semantics. One is a structural semantics, where they are interpreted with respect to a ternary composition relation (or ‘Merge’, as it is called in generative grammar). The truth conditions for

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this interpretation are given in [2]; one finds the basic soundness/completeness results in [3]. The second interpretation is a computational one, along the lines of the Curry-Howard formulas-as-types program. Under this second interpretation, Lambek derivations are associated with a linear (and structure-sensitive) sublanguage of the lambda terms one obtains for proofs in positive Intuitionistic logic. The slashes /, \ here are seen as directional implications; elimination of these operations corresponds to function application, introduction to lambda abstraction.

\[
x \vdash A \otimes B \text{ iff } \exists yz.R_{\otimes}xyz \text{ and } y \vdash A \text{ and } z \vdash B
\]

\[
y \vdash C/B \text{ iff } \forall xz.(R_{\otimes}xyz \text{ and } z \vdash B) \text{ implies } x \vdash C
\]

\[
z \vdash A\backslash C \text{ iff } \forall xy.(R_{\otimes}xyz \text{ and } y \vdash A) \text{ implies } x \vdash C
\]

The original Lambek calculus, like its predecessors the Ajdukiewicz/Bar Hillel (AB) calculi, and later systems such as Combinatory Categorial Grammar (CCG), adequately deals with linguistic subcategorization or valency. It greatly improves on AB and CCG systems in fully supporting hypothetical reasoning: the bidirectional implications of the residuation laws are fully symmetric with respect to putting together larger phrases out of their subphrases, and taking apart compound phrases in their constituent parts. AB systems lack the second feature completely; the combinatory schemata of CCG provide only a weak approximation. Consequences of hypothetical reasoning are the theorems of type lifting and argument lowering of (3) below; type transitions of this kind have played an important role in our understanding of natural language semantics.

\[
A \rightarrow B/(A\backslash B) \quad (B/(A\backslash B))\backslash B \rightarrow A\backslash B \tag{3}
\]

It is ironic that precisely in the hypothetical reasoning component the Lambek grammars turn out to be deficient. As one sees in [3], hypothetical reasoning typically involves higher order types, where a slash occurs in a negative environment as in the schema (4) below. Given Curry-Howard assumptions, the associated instruction for meaning assembly has an application, corresponding to the elimination of the main connective /, and an abstraction, corresponding to the introduction of the embedded \.

\[
C/(A\backslash B) \quad (M \lambda x^A.N^B)^C \tag{4}
\]

The minimal Lambek calculus falls short in its characterization of which A-type hypotheses are ‘visible’ for the slash introduction rule: for the residuation rules to be applicable, the hypothesis has to be structurally peripheral (left peripheral for \, right peripheral for /). One can distinguish two kinds of problems.

**Displacement.** The A-type hypothesis occurs internally within the domain of type B. Metaphorically, the functor C/(A\backslash B) seems to be displaced from the site of the hypothesis. Example: wh ‘movement’.

**Non-local semantic construal.** The functor (e.g. C/(A\backslash B)) occupies the structural position where in fact the A-type hypothesis is needed, and realizes its semantic effect at a higher structural level. (The converse of the above.) Example: quantifying expressions.