Continuation Semantics for Symmetric
Categorial Grammar

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Abstract. Categorial grammars in the tradition of Lambek [1,2] are
asymmetric: sequent statements are of the form \( \Gamma \Rightarrow A \), where the succe-
dent is a single formula \( A \), the antecedent a structured configuration of
formulas \( A_1, \ldots, A_n \). The absence of structural context in the succedent
makes the analysis of a number of phenomena in natural language
semantics problematic. A case in point is scope construal: the different pos-
sibilities to build an interpretation for sentences containing generalized
quantifiers and related expressions. In this paper, we explore a symmet-
ric version of categorial grammar based on work by Grishin [3]. In addi-
tion to the Lambek product, left and right division, we consider a dual
family of type-forming operations: coproduct, left and right difference.
Communication between the two families is established by means of
structure-preserving distributivity principles. We call the resulting system
\( \text{LG} \). We present a Curry-Howard interpretation for \( \text{LG}(\mathbin{\vee}, \mathbin{\wedge}, \mathbin{\odot}, \mathbin{\oslash}) \) deri-
vations. Our starting point is Curien and Herbelin’s sequent system for \( \lambda \mu 
\) calculus [4] which capitalizes on the duality between logical implication
(i.e. the Lambek divisions under the formulas-as-types perspective) and
the difference operation. Importing this system into categorial grammar
requires two adaptations: we restrict to the subsystem where linearity con-
ditions are in effect, and we refine the interpretation to take the left-right
symmetry and absence of associativity/commutativity into account. We
discuss the continuation-passing-style (CPS) translation, comparing the
call-by-value and call-by-name evaluation regimes. We show that in the lat-
ter (but not in the former) the types of \( \text{LG} \) are associated with appropriate
denotational domains to enable a proper treatment of scope construal.

1 Background

Lambek-style categorial grammars offer an attractive computational perspective
on the principle of compositionality: under the Curry-Howard interpretation,

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derivations are associated with instructions for meaning assembly. In natural language semantics, scope construal of generalized quantifier expressions presents an ideal testing ground to bring out the merits of this approach. Scope construal exemplifies a class of phenomena known as \textit{in situ} binding. An \textit{in situ} binder syntactically occupies the position of a phrase of type $A$; semantically, it binds an $A$-type variable in that position within a context of type $B$, producing a value of type $C$ as a result. The inference pattern of (1) (from [5]) schematically captures this behaviour in the format of a sequent rule. The challenge is to solve the equation for the type alias $q(A,B,C)$ in terms of the primitive type-forming operations.

$$\frac{\Delta[x : A] \Rightarrow N : B}{\Gamma[y : C] \Rightarrow M : D} \quad \frac{\Gamma[\Delta[z : q(A,B,C)]] \Rightarrow M[y := (\lambda x.N)] : D }{.}$$

It is a poignant irony that precisely in the area of scope construal, the performance of the original Lambek calculus (whether in its associative or non-associative incarnation) is disappointing. For a sentence-level generalized quantifier (GQ) phrase, we have $A = np$, $B = C = s$ in (1). The type-forming operations available to define $q(np,s,s)$ are the left and right slashes. A first problem is the lack of \textit{type uniformity}. Given standard modeltheoretic assumptions about the interpretation of the type language, an assignment $s/(np\backslash s)$ to a GQ phrase is associated with an appropriate domain of interpretation (a set of sets of individuals), but with such a type a GQ is syntactically restricted to subject positions: for phrase-internal GQ occurrences, context-dependent extra lexical type assignments have to be postulated. Second, this lexical ambiguity strategy breaks down as soon as one considers \textit{non-local} scope construal, where the distance between the GQ occurrence and the sentential domain where it establishes its scope can be unbounded.

The solutions that have been proposed in the type-logical literature we consider suboptimal. The type-shifting approach of Hendriks [6] and the multimodal accounts based on wrapping operations of Morrill and co-workers [7,8] each break the isomorphic relation between derivations and terms that is at the heart of the Curry-Howard interpretation. Hendriks introduces a one-to-many dichotomy between syntactic and semantic derivations. Morrill makes the opposite choice: a multiplicity of syntactically distinct implicational operations which collapse at the semantic level.

The approach we develop in the sections below sticks to the \textit{minimal} categorial logic: the pure logic of residuation. We overcome the expressive limitations of the Lambek calculi by lifting the single succedent formula restriction and move to a symmetric system where the Lambek connectives (product, left and right division) coexist with a dual family (coproduct, right and left difference). The communication between these two families is expressed in terms of Grishin’s [3] distributivity principles. Figure 1 schematically presents the outline of the paper. In $\S$ we present $\text{LG}$ in algebraic format and discuss the symmetries that govern the vocabulary of type-forming operations. In $\S$ we present a ‘classical’ term language for the $\text{LG}$ type system, and we discuss how a term $\tau$ of type $A$ is obtained as the Curry-Howard image of an $\text{LG}$ sequent derivation $\pi$. In $\S$ we then study the