2 Axion Cosmology

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Abstract. The cosmological properties of axions are reviewed. We discuss the axions produced by thermal processes in the early universe, the evolution of the average axion field between the Peccei-Quinn and QCD phase-transitions, the domain-wall problem and its possible resolutions, the population of cold axions produced by vacuum realignment, string decay and domain wall decay, and, finally, axion miniclusters and axion isocurvature perturbations.

For background information on the strong CP problem and on laboratory and astrophysical axion constraints, the reader is referred to Chaps. 1 and 3. In this chapter, we will be concerned only with the cosmological properties of axions. We merely mention that different authors may use different definitions of the axion decay constant f_a. We define normalization by the action density for QCD plus an axion

$$L_{\mathrm{QCD}+a} = -\frac{1}{4} G^b_{\mu\nu} G^{b\mu\nu} + \frac{1}{2} \partial_\mu a \, \partial^\mu a + \sum_q \bar{q} \left(\mathrm{i}\gamma^\mu \partial_\mu - m_q \right) q + \frac{g_\mathrm{s}^2}{32\,\pi^2} \left(\theta + \frac{a}{f_a} \right) G^b_{\mu\nu} \tilde{G}^{b\mu\nu} \, , \tag{2.1}$$

where a is the axion field before mixing with the η and π^0 mesons. Equation (2.1) uses standard notation for the chromomagnetic field strength tensor G, the strong coupling constant g_s, and the quark fields q. The axion mass, after mixing with the η and π^0 mesons, is given in terms of f_a by

$$m_a \simeq 6\,\mu\mathrm{eV} \left(\frac{10^{12}\,\mathrm{GeV}}{f_a} \right) . \tag{2.2}$$

The axion-decay constant is related to the magnitude v_a of the vacuum expectation value that breaks the $U(1)_\mathrm{PQ}$ symmetry by $f_a = v_a/N$. N is an integer characterizing the color anomaly of $U(1)_\mathrm{PQ}$, and $N = 6$ in the original Peccei-Quinn-Weinberg-Wilczek axion model. All axion couplings are inversely proportional to f_a.

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2.1 Thermal Axions

Axions are created and annihilated during interactions among particles in the primordial soup. Let us call the population of axions established as a result of such processes "thermal axions", to distinguish them from the population of "cold axions" which we discuss later. The number density $n_a^{\rm th}(t)$ of thermal axions solves the Boltzmann equation [1]

$$\frac{{\rm d}n_a^{\rm th}}{{\rm d}t} + 3Hn_a^{\rm th} = \Gamma \left(n_a^{\rm eq} - n_a^{\rm th}\right) \ , \tag{2.3}$$

where

$$\Gamma = \sum_i n_i \langle \sigma_i v \rangle \tag{2.4}$$

is the rate at which axions are created and annihilated. $H(t)$ is the Hubble expansion rate and

$$n_a^{\rm eq} = \frac{\zeta(3)}{\pi^2} T^3 \tag{2.5}$$

is the number density of axions at thermal equilibrium, where $\zeta(3) = 1.202\ldots$ is the Riemann zeta function of argument 3. In (2.4), the sum is over processes of the type $a + i \leftrightarrow 1 + 2$, where 1 and 2 are other particle species, n_i is the number density of particle species i, σ_i is the corresponding cross section, and $\langle \ldots \rangle$ indicates averaging over the momentum distributions of the particles involved.

Unless unusual events are taking place, $T \propto R^{-1}$ where $R(t)$ is the scale factor, and (2.5) implies, therefore,

$$\frac{{\rm d}n_a^{\rm eq}}{{\rm d}t} + 3Hn_a^{\rm eq} = 0 \ . \tag{2.6}$$

Combining (2.6) and (2.3), one obtains

$$\frac{\rm d}{{\rm d}t}\left[R^3 (n_a^{\rm th} - n_a^{\rm eq})\right] = -\Gamma R^3 \left(n_a^{\rm th} - n_a^{\rm eq}\right) \ . \tag{2.7}$$

This equation implies that a thermal distribution of axions is approached exponentially fast whenever the condition

$$\Gamma > H \tag{2.8}$$

is satisfied. So, we have a thermal population of axions today, provided the inequality (2.8) prevailed for a few expansion times at some point in the early universe and the thermal population of axions thus established did not subsequently get diluted away by inflation or some other cause of huge entropy release.

The least model-dependent processes for thermalizing axions in the early universe are (a) $a + q(\bar{q}) \leftrightarrow g + q(\bar{q})$, (b) $a + g \leftrightarrow q + \bar{q}$, and (c) $a + g \leftrightarrow g + g$.