Steiner Forests on Stochastic Metric Graphs

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Abstract. We consider the problem of connecting given vertex pairs over a stochastic metric graph, each vertex of which has a probability of presence independently of all other vertices. Vertex pairs requiring connection are always present with probability 1. Our objective is to satisfy the connectivity requirements for every possibly materializable subgraph of the given metric graph, so as to optimize the expected total cost of edges used. This is a natural problem model for cost-efficient Steiner Forests on stochastic metric graphs, where uncertain availability of intermediate nodes requires fast adjustments of traffic forwarding. For this problem we allow a priori design decisions to be taken, that can be modified efficiently when an actual subgraph of the input graph materializes. We design a fast (almost linear time in the number of vertices) modification algorithm whose outcome we analyze probabilistically, and show that depending on the a priori decisions this algorithm yields 2 or 4 approximation factors of the optimum expected cost. We also show that our analysis of the algorithm is tight.

1 Introduction

We consider the problem of laying out routes that connect simultaneously given source-destination vertex pairs over a metric graph $G_0(V_0, E_0)$. Vertices of the metric graph $G_0$ other than the sources and destinations may be used, but we are uncertain of their availability, in that each such vertex is present with some probability independently of all other vertices. Sources and destinations are present with probability 1. Our objective is to take some a priori decisions regarding the layout of required routes, so as to be able to come up with feasible routes for every possibly materializable subgraph $G_1(V_1, E_1)$, $V_1 \subseteq V_0$, of $G_0$, and minimize the expected total cost of edges used over the distribution of all such subgraphs. This is the well known Steiner Forest problem, defined over a stochastic metric graph $G_0$.

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A brute-force way to cope with this problem is to precompute a feasible and approximate (or maybe optimum) solution for every possible subgraph of $G_0$ that may materialize, and apply an appropriate solution when the subgraph actually appears. In principle there need not be a constraint on the computational effort applied for taking a priori decisions, as long as they support fast response to the actually materialized data. In this light however, we require that such a response should be of strictly lower complexity compared to the a priori computational effort. A straightforward pattern for implementing this setting is for example to compute an optimum a priori solution over $G_0$, and if this solution is not feasible for the materialized subgraph $G_1$, use a polynomial-time approximation algorithm to obtain a completely different feasible solution for $G_1$. On the other hand, a natural challenge is to design such an efficient response strategy (algorithm), that can be supported by polynomial-time computable a priori decisions. In this paper we design and analyze such a strategy for repairing an a priori polynomial-time computable feasible solution for $G_0$, so as to render it feasible for $G_1$. We show that this strategy also approximates the optimum expected cost over all materializable subgraphs $G_1$.

The problem model we consider finds natural application in networks, where uncertain availability of intermediate nodes requires fast adjustments of traffic forwarding. The Steiner Forest problem is a well-known NP-hard multicommodity network design problem (even in metric graphs), generalizing the Steiner tree problem, and the only known approximation algorithm yields approximation factor 2 and was analyzed in \cite{1,8} (see also \cite{18}). Recent years have seen a detailed study and sensitivity analysis of this algorithm, mainly in the context of Stochastic Network Design, which owes its roots to Stochastic Programming \cite{5,4}, where some elements of the input data set to an optimization problem are associated to a distribution describing their probability of occurence. Stochastic Programming was introduced by the seminal work of Dantzig \cite{5} and thereafter has evolved into an independent discipline of Operations Research that handles uncertainty in optimization problems by usage of probabilities, statistics and mathematical programming (see \cite{4} for a description of the field). We refer the reader to \cite{11,7} for approximation results on Stochastic Steiner Forest models and to \cite{13,12,10,6,9} for additional recent approximation results on stochastic network design problems in general.

Our work is mostly related to the framework of Probabilistic Combinatorial Optimization, introduced in \cite{2,14}, where repairing strategies as the one described previously are analyzed probabilistically, so that the expected cost of their outcome can be computed efficiently (this ensures that the problem of taking a priori decisions for a particular strategy belongs to class NPO). Several network design problems have been treated in the probabilistic combinatorial optimization framework, including minimum coloring \cite{17}, maximum independent set \cite{16}, longest path \cite{15}, and minimum spanning tree \cite{3}. Apart from probabilistic analysis of repairing strategies, results in \cite{17,16} also include derivation of approximability properties.