Study of 3D Dynamics of Gravitating Systems Using Supercomputers: Methods and Applications

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Abstract. We describe parallel numerical code for solving problems of stellar dynamics. The code is based on numerical solving of Poisson and Vlasov equations in cylindrical coordinates using particle-in-cells method. The code is designed for use on supercomputers with distributed memory. We consider different possible strategies of parallelization according to initial technical parameters of numerical methods and physical conditions of the model. We present results of numerical simulations for the following problems of stellar dynamics: investigation of influence of central potential on the vertical motions of thin gravitating disk; stability of uniform sphere with anisotropic distribution of velocity; numerical approximation of equilibrium states of gravitating systems.

1 Introduction

Problems of stellar dynamics — investigation of stellar systems formation, their equilibrium and stability, appearance of spirals and bars — require to solve N-body problem in self-consistent gravitational field [1]. Its mathematical model consists of collisionless Vlasov equation for distribution function of matter (hereinafter, DF) and Poisson equation for gravitational potential. Numerical solving is based on particle-in-cells method [2] (also called particle-mesh).

Complexity of this numerical model is conditioned by three-dimensions and non-stationarity of the problem. It’s required to compute individual motions of huge number of particles, to solve 3D Poisson equation and to store 3D mesh functions of potential, gravitational forces and density of matter as 3D arrays in computer’s RAM. At the same time the number of particles and nodes of

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the mesh should be sufficient to provide reliability of the simulation results. That’s why numerical simulations for considered class of problems are close to computer’s capabilities. Hence there is a strong requirement to develop effective parallel algorithms and to employ supercomputers.

Mentioned difficulties can be partially overcome with the help of quasi-3D model \cite{3}, special approximation of 3D model, which neglects vertical motions of the matter (however it’s still needed to solve 3D Poisson equation). This approximation is especially useful to overcome problem with storing 3D data, because on each time step only values of mesh functions in plane \( z = 0 \) (where matter has non-zero density) are needed. It seems, that quasi-3D approximation is suitable in the presence of massive central gravitational field and initial DF in the form of thin disk, that is the case of circumstellar disk model. However for the large class of problems, such as investigation of globular clusters and systems with distinct vertical motions of matter or non-uniform vertical structure, completely 3D model must be studied.

In the present paper we describe parallel numerical algorithms for investigation of 3D dynamics of gravitating systems. We consider possible approaches to the parallelization of numerical methods according to their technical parameters (number of mesh nodes and particles) and initial physical conditions of the problem. With the help of implemented parallel code we are able to perform numerical simulations for important problems of stellar dynamics. We present some applications:

- investigation of influence of central potential on the vertical motions of thin gravitating disk,
- stability of uniform sphere with anisotropic distribution of velocity,
- approach to study equilibrium states of gravitating systems.

2 Mathematical Model of 3D Dynamics of Gravitating Systems

The foundation of numerical model of 3D dynamics of gravitating systems is collisionless Vlasov equation for DF and Poisson equation for self-consistent gravitational potential \cite{1}.

Collisionless Vlasov equation has the following form:

\[
\frac{\partial f}{\partial t} + \mathbf{u} \frac{\partial f}{\partial \mathbf{r}} - \nabla \Phi \frac{\partial f}{\partial \mathbf{u}} = 0,
\]

where \( f(t, \mathbf{r}, \mathbf{u}) \) is time-dependent DF of coordinates \( \mathbf{r} \) and velocities \( \mathbf{u} \). Gravitational potential satisfies Poisson equation, which has the following form in chosen cylindrical coordinates:

\[
\frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial \Phi}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 \Phi}{\partial \phi^2} + \frac{\partial^2 \Phi}{\partial z^2} = 4\pi G \rho.
\]

\footnote{E.g. there are very few supercomputers in the world allowing to carry out experiments with the number of particles equal to number of stars in Galaxy \( \sim 10^{11} \).}