Time Series Analysis of Nonstationary Data in Encephalography and Related Noise Modelling

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Abstract

In this report, statistical time series analysis of nonstationary EEG/MEG data is proposed. The signal is investigated as a stochastic process, and approximated by a set of deterministic components contaminated by the noise which is modelled as a parametric autoregressive process. Separation of the deterministic part of time series from stochastic noise is obtained by an application of matching pursuit algorithm combined with testing for the residuum's weak stationarity (in mean and in variance) after each iteration. The method is illustrated by an application to simulated nonstationary data.

1. Introduction

In brain evoked activity measured by means of EEG/MEG, one can observe time-dependent changes of its various characteristics like amplitude and frequency, as well as the contaminating noise. For this reason, it is necessary to use the analysis methods designed for nonstationary signals, since the standard EEG/MEG methodology based on signal averaging and simple spectral analysis is insufficient. Time-frequency estimation methods such as short-time Fourier transform, Wigner distribution, or discrete and continuous wavelet transform are very useful, yet, statistically inefficient. They also have some inherent limitations. Thus, the representation of the evoked-response generative process given by these methods is incomplete. In this research, EEG/MEG signal is investigated as a stochastic process which can be decomposed to a set of deterministic functions representing its nonstationarity and stationary residua. For modelling the stochastic EEG/MEG noise, statistical time series analysis methods are used.
2. Statistical time series analysis

A time series (TS) model for the observed data \( \{z(t)\} \) is a specification of the joint distributions (or possibly of only the means and covariances) of a sequence of random variables \( \{Z_t\} \), with a realization denoted by \( \{z(t)\} \) [1]. In a short form, an additive TS model can expressed by the sum of deterministic \( d(t) \) and stochastic \( l(t) \) components:

\[
z(t) = d(t) + l(t)
\]  

(1)

Off course, there are many possible examples for this kind of a model, i.e. \( d(t) \) can be a linear trend, a seasonal (periodic) function, or a sum of them, and \( l(t) \) can be a set of observations of any (stationary or nonstationary) random variable. Let us take times series generated by an additive stochastic process given by (2), which is the sum of \( m \in N \) sine waves or other non-commensurable periodic functions (or commensurable but with a period much longer than the periods of its particular components) \( s(t) \) and a stationary noise \( e(t) \).

\[
z(t) = \sum_{i=0}^{m} s^i(t) + e(t)
\]  

(2)

Modelling of a process requires all the deterministic functions to be removed at first. It can be achieved by the preceding estimation of these functions or by differencing the series. There is a lot of helpful examining tools (i.e. statistical tests and spectral analysis methods (spectral density, periodogram) based on the Fourier transform) and estimation techniques (i.e. maximum likelihood method). Next, stochastic, stationary residua can be diagnosed (on the basis of the sample autocorrelation function (ACF) and sample partial autocorrelation function (PACF), and using some statistical tests) and an adequate parametric model of them (autoregressive (AR) and/or moving average (MA) for example) can be constructed [1].

3. Time series de-trending by matching pursuit

Unfortunately, EEG/MEG signals are nonstationary, and so the characteristics of \( s^i(t) \) components in Equation (2) varies in time (thus we can not assume that each \( s(t) \) is periodic). In consequence, a typical time series decomposition, based on the Fourier transform, disappoints in this case.