14 Interaction Between Atoms and Quantized Fields

In Chaps. 1–12, we consider the interactions of atoms with optical fields in the semiclassical approximation, a treatment which gives good agreement with many experiments. Nevertheless, several significant experiments show strong disagreements with such semiclassical theories. The present chapter treats the atom-field interaction fully quantum mechanically, providing a basic understanding of spontaneous emission and laying the foundations for the treatment of more advanced problems in quantum optics such as resonance fluorescence, squeezed states and the laser linewidth that are developed in the remaining chapters in the book. Basically all we need to do in laying the foundations for these treatments is to combine the knowledge we have gained from the semiclassical theory of the atom-field interactions with the quantized field treatment of Chap. 13.

It would be misleading to say that this marriage is easy. There are some nasty infinities floating around that one has to argue away. In principle this is hard to do. Quantum electrodynamics (QED), which handles electron-photon interactions for arbitrary electron velocities and field frequencies, has developed techniques such as renormalization to solve these problems. There are numerous excellent books on the subject, such as, e.g., Itzykson and Zuber (1980). There are also subtle difficulties associated with gauge invariance which are of direct relevance to a number of quantum optics applications and have been the subject of heated debate in the past. Cohen-Tannoudji et al. (1989) discuss these problems in great detail in the framework on nonrelativistic QED. Our ambition in this book is much more limited. We restrict our considerations to nonrelativistic velocities and low (optical) frequency photons. In these limits, we further make plausible assumptions that sidestep the real problems with QED.

We start with the relatively simple problem of an atom coupled to a single quantized mode of the field. This problem is called the Jaynes-Cummings model, which is perhaps ironic, since after working on the problem, Jaynes has steadfastly championed atom-field theories that use classical fields. We introduce the “dressed-atom” picture, which can be very useful in explaining how elementary phenomena take place. In particular, we gain a simple understanding of the “light shift” and set up the understanding of the three peaks of strong field resonance fluorescence. Section 14.2 discusses the dynamics of
the atom-field model for various states of the field. We obtain an elementary picture of spontaneous and stimulated emission and absorption, and briefly discuss the “Cummings collapse” and revivals due to the quantum granularity of the field. Section 14.3 derives the spontaneous emission decay rate both using the Fermi Golden Rule and using the more generally accurate Weisskopf-Wigner theory. This problem is important in its own right and provides an excellent example of system-reservoir interactions, discussed in greater detail in Chap. 15.

14.1 Dressed States

In Chap. 3 we show that the interaction Hamiltonian between an atom and a classical field is given in the dipole approximation by

\[ \mathcal{V} = -\epsilon \mathbf{r} \cdot \mathbf{E}, \]  

(14.1)

where \( \mathbf{E} \) is the electric field and \( \epsilon \mathbf{r} \) is the atomic dipole moment operator. The form of the interaction energy remains the same for quantized fields, but \( \mathbf{E} \) becomes the electric field operator discussed in Chap. 13. For the single mode field (13.13), the interaction Hamiltonian (14.1) becomes

\[ \mathcal{V} = \hbar \left( a + a^\dagger \right) \left( g \sigma_+ + g^* \sigma_- \right), \]  

(14.2)

where \( \sigma_+ \) and \( \sigma_- \) are the Pauli spin-flip matrices (3.128) and the electric-dipole matrix element

\[ g = \frac{\nu \mathcal{E} \Omega}{2 \hbar} \sin Kz. \]  

(14.3)

With the two-level unperturbed Hamiltonian \( \frac{1}{2} \hbar \omega \sigma_z \) [see (3.128)] and the free-field Hamiltonian (13.6) (without the zero point energy), we obtain the total atom-field Hamiltonian

\[ \mathcal{H} = \frac{1}{2} \hbar \omega \sigma_z + \hbar \Omega a^\dagger a + \hbar \left( a + a^\dagger \right) \left( g \sigma_+ + g^* \sigma_- \right). \]  

(14.4)

Without loss of generality for two-level systems at rest, we can choose the atomic quantization axis such that the matrix element \( g \) is real. One of the basic approximations in the theory of two-level atoms is the rotating wave approximation. We can understand how this approximation works with quantized fields by considering the various terms in the interaction energy (14.2). \( a \sigma_+ \) corresponds to the absorption of a photon and the excitation of the atom from the lower state \( |b\rangle \) to the upper state \( |a\rangle \). Conversely, \( a^\dagger \sigma_- \) describes the emission of a photon and the de-excitation of the atom. These combinations correspond to those kept in the rotating wave approximation. To see how the remaining two pairs, \( a \sigma_- \) and \( a^\dagger \sigma_+ \) are dropped in this approximation, consider the free evolution \( (g = 0) \) of these operators in