15 System-Reservoir Interactions

The Weisskopf-Wigner theory of spontaneous emission of Chap. 14 is an example of a general class of problems involving the coupling of a small system to a large system. In that case the small system is the atom and the large system is the continuum of modes of the electromagnetic field. When computing the atomic decay rate, we were not interested in the field itself, but only on its effect on the atomic dynamics. Thus we never explicitly computed the field dynamics. Our theory leads to an irreversible decay of the upper state population. At first this should come as a surprise, since our starting equation (the Schrödinger equation) is reversible. Irreversibility results from two main approximations: 1) the assumption that the probability amplitude $C_{a0}(t)$ varies little during the time interval defined by the inverse bandwidth of the continuum of modes of the electromagnetic field, and 2) the replacement of the remaining nonlocal time integration in (14.58) by a $\delta$-function. These choices comprise the Weisskopf-Wigner approximation.

Similar situations occur repeatedly in physics in general, and in quantum optics in particular, and always lead to an irreversible decay of the small system. In fact, each time one wishes to describe properly irreversible damping and decoherence in quantum mechanics, one does so by coupling the small system under study to a large, broad-band system which typically remains in thermal equilibrium. For this reason, the large system is usually called a bath, or a reservoir. It is of considerable importance to develop a general formalism to handle this problem.

Just as there are two fundamental ways to treat a general quantum mechanics problem, the Heisenberg and the Schrödinger pictures, there are also two basic ways to tackle system-reservoir interactions. The first one is based on the Schrödinger picture and leads to the so-called master equation. We discuss it in Sect. 15.1 and use it in Sect. 16.4 on resonance fluorescence and in Sect. 17.2 on the quantum theory of multiwave mixing. In Sect. 15.2, we show how an expansion of the system density operator in a basis of coherent states permits us to transform the master equation into a Fokker-Planck equation. The Heisenberg approach is presented in Sect. 15.3 and leads to the introduction of quantum noise operators giving the description of the problem a flavor reminiscent of the Langevin approach to stochastic problems in classical physics. Section 15.4 discusses the method of Monte Carlo.
wave functions, which permits to unravel the master equation into “quantum trajectories” of considerable intuitive appeal and numerical convenience. Section 15.5 explains the quantum regression theorem and applies it to the evaluation of two-time correlation functions such as appear in spectrum calculations in resonance fluorescence and in the generation of squeezed states.

The material of this chapter is rather technical, and our experience has been that a general presentation tends to mask the physics involved in deriving the results. We prefer therefore to sacrifice generality and concentrate on an illustration of the theory for the case of a simple harmonic oscillator coupled to a bath of harmonic oscillators. This model system is described by the Hamiltonian

\[ H = H_s + H_r + V, \]  

(15.1)

where

\[ H_s = \hbar \Omega a^\dagger a \]  

(15.2)

is the unperturbed Hamiltonian of the small system,

\[ H_r = \sum_j \hbar \omega_j b_j^\dagger b_j \]  

(15.3)

the unperturbed Hamiltonian of the reservoir, consisting of a very large number of harmonic oscillators, and

\[ V = \hbar \sum_j (g_j a^\dagger b_j + g_j^* b_j^\dagger a) \]  

(15.4)

is a model for the system-reservoir interaction. The elementary exchange of energy between system and bath is thus assumed to consists of the simultaneous creation of a quantum of excitation of the system with annihilation of a quantum in the \( j \)th mode of the bath, or the reverse process.

Different problems require of course different model Hamiltonians. For instance, Sect. 15.1 concludes with a corresponding discussion for a resonant bath of two-level atoms and the remaining chapters in the book deal with related problems. The general behavior of the system is however not very sensitive to the explicit form of \( H_r \), provided that it meets some general requirements, the most important one being that it has a broadband spectrum, and hence a very short correlation time. The specific model defined by (15.1–15.4) is a good model of coupling to the continuum of modes of the electromagnetic field, of phonons in a crystal, etc., which is one reason why we consider it here. The other reason is that harmonic oscillators are the simplest quantum systems, and make our lives particularly easy.

The Hamiltonian (15.1), together with initial conditions, completely defines our problem. We suppose that at the initial time \( t_0 \) the small system is described by a density operator \( \rho_s(t_0) \), where the subscript \( s \) indicates that this is the system’s density operator. Of course, \( \text{tr}_s\{\rho_s(t_0)\} = 1 \), where \( \text{tr}_s \) means “trace over the system”. In contrast we suppose that the reservoir is