Chapters 1, 2 describe the interaction of radiation with matter in terms of a phenomenological classical polarization \( P \). The question remains as to when this approach is justified and what to do when it isn’t. Unexcited systems interacting with radiation far from the system resonances can often be treated purely classically. The response of the system near and at resonance often deviates substantially from the classical descriptions. Since the laser itself and many applications involve systems near atomic (or molecular) resonances, we need to study them with the aid of quantum mechanics.

In preparation for this study, this chapter reviews some of the highlights of quantum mechanics paying particular attention to topics relevant to the interaction of radiation with matter. Section 3.1 introduces the wave function for an abstract quantum system, discusses the wave function’s probabilistic interpretation, its role in the calculation of expectation values, and its equation of motion (the Schrödinger equation). Expansions of the wave function in various bases, most notably in terms of energy eigenstates, are presented and used to convert the Schrödinger partial differential equation into a set of ordinary differential equations. The Dirac notation is reviewed and used to discuss the state vector and how the state vector is related to the wave function. System time evolution is revisited with a short review of the Schrödinger, Heisenberg and interaction pictures.

In Chaps. 4–12, we are concerned with the interaction of classical electromagnetic fields with simple atomic systems. Section 3.2 lays the foundations for these chapters by discussing wave functions for atomic systems and studying their evolution under the influence of applied perturbations. Time dependent perturbation theory and the rotating wave approximation are used to predict this evolution in limits for which transitions are unlikely. The Fermi Golden Rule is derived. Section 3.3 deals with a particularly simple atomic model, the two-level atom subject to a resonant or nearly resonant classical field. We first discuss the nature of the electric-dipole interaction and then use the Fermi Golden Rule to derive Einstein’s \( A \) and \( B \) coefficients for spontaneous and stimulated emission. We then relax the assumption that the interaction is weak and derive the famous Rabi solution.

In Chaps. 13–19, we discuss interactions for which the electromagnetic field as well as the atoms must be quantized. In particular, Chap. 13 shows
that electromagnetic field modes are described mathematically by simple harmonic oscillators. In addition, these oscillators can model the polarization of certain kinds of media, such as simple molecular systems. In preparation for such problems, Sect. 3.4 quantizes the simple harmonic oscillator. The section writes the appropriate Hamiltonian in terms of the annihilation and creation operators, and derives the corresponding energy eigenstates.

This chapter is concerned with the quantum mechanics of single systems in pure states. Discussions of mixtures of systems including the decay phenomena and excitation mechanisms that occur in lasers and their applications are postponed to Chap. 4 on the density matrix.

3.1 Review of Quantum Mechanics

According to the postulates of quantum mechanics, the best possible knowledge about a quantum mechanical system is given by its wave function \( \psi(r, t) \). Although \( \psi(r, t) \) itself has no direct physical meaning, it allows us to calculate the expectation values of all observables of interest. This is due to the fact that the quantity

\[
\psi(r, t)^* \psi(r, t) \, d^3r
\]

is the probability of finding the system in the volume element \( d^3r \). Since the system described by \( \psi(r, t) \) is assumed to exist, its probability of being somewhere has to equal 1. This gives the normalization condition

\[
\int \psi(r, t)^* \psi(r, t) \, d^3r = 1, \quad (3.1)
\]

where the integration is taken over all space.

An observable is represented by a Hermitian operator \( O(r) \) and its expectation value is given in terms of \( \psi(r, t) \) by

\[
\langle O \rangle(t) = \int d^3r \psi^*(r, t) O(r) \psi(r, t).
\]  
(3.2)

Experimentally this expectation value is given by the average value of the results of many measurements of the observable \( O \) acting on identically prepared systems. The accuracy of the experimental value for \( \langle O \rangle \) typically depends on the number of measurements performed. Hence enough measurements should be made so that the value obtained for \( \langle O \rangle \) doesn’t change significantly when still more measurements are performed. It is crucial to note that the expectation value (3.2) predicts the average from many measurements; in general it is unable to predict the outcome of a single event with absolute certainty. This does not mean that quantum mechanics in other ways is unable to make some predictions about single events.