Cyclic Railway Timetabling: 
A Stochastic Optimization Approach

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Abstract. Real-time railway operations are subject to stochastic disturbances. However, a railway timetable is a deterministic plan. Thus a timetable should be designed in such a way that it can absorb the stochastic disturbances as well as possible. To that end, a timetable contains buffer times between trains and supplements in running times and dwell times. This paper first describes a stochastic optimization model that can be used to find an optimal allocation of the running time supplements of a single train on a number of consecutive trips along the same line. The aim of this model is to minimize the average delay of the train. The model is then extended such that it can be used to improve a given cyclic timetable for a number of trains on a common railway infrastructure. Computational results show that the average delay of the trains can be reduced substantially by applying relatively small modifications to the timetable. In particular, allocating the running time supplements in a different way than what is usual in practice can be useful.

1 Introduction

Punctuality of railway services is a highly important issue, since punctuality is considered as one of its main performance indicators. In the Netherlands, punctuality is defined as the percentage of trains that arrive with a delay of less than 3 minutes at one of the larger railway stations. In several other countries, a 5 minute margin is used, or only the delays at the end points of a line are taken into account. Delays of trains occur since real-time railway operations are subject to stochastic disturbances. However, the underlying railway timetable is a deterministic plan. Therefore, the stochastic disturbances in the operations should be taken into account in the design of a timetable as well as possible: the timetable
should be robust. In order to cope with the disturbances in the real-time operations, a timetable contains buffer times between trains and supplements in the running times and in the dwell times of the trains.

Many authors addressed the analysis and the improvement of the punctuality of railway services: several relevant models have been developed to that end. The main examples of these models are the following: (i) simulation models (see Bergmark [1], König [10], Middelkoop and Bouwman [13], and Wahlborg [24]), (ii) Max-Plus models (see Goverde [4], De Kort [11], and Soto Y Koelemeijer et al. [21]), and (iii) analytical models (see Carey [3], Higgins and Kozan [7], and Huisman and Boucherie [8]). Other relevant literature on stochastic methods for the improvement of railway timetables is Hallowell and Harker [6], Schwanhäuser [19], Mühlhans [14], and Petersen and Taylor [17]. However, a drawback of the existing models is that they are mainly evaluation models and that, based on these models, optimization of a timetable can only be achieved by trial-and-error: the timetable is modified and then the evaluation model is used afterwards to evaluate the effect of the modification. If necessary, these steps are repeated.

In contrast with the existing models, the current paper describes a stochastic optimization model that can be used to modify a given cyclic timetable and, at the same time, to evaluate the modified timetable by operating a number of realizations of the trains in the timetable. We refer to Birge and Louveaux [2] for more information on stochastic optimization. In our model, the trains are operated as much as possible according to the modified timetable, but subject to external stochastic disturbances. The main criterion that is used to modify the timetable is minimization of the average delay of the trains. Note that other criteria can be handled as well. The structure of the model is such that it is a symbiosis of a timetabling model and a simulation model.

The first model in this paper generates a timetable for a single train that is operated under stochastic external disturbances on a number of consecutive trips along the same line. Here a trip is a movement of a train from one station to the next. The model is used to allocate a fixed total amount of running time supplement to the consecutive trips such that the average delay of the train is minimal. The model is then extended to be applicable in a more complex situation where several trains are operated according to a given cyclic timetable and on a common railway infrastructure. These trains are also operated under stochastic external disturbances. The extended model is used here to improve the timetable with respect to the average delay of the trains by re-allocating buffer times and time supplements. The application of the extended model to a practical case shows that, within the model, the modification of a given timetable may lead to a substantial reduction of the average delay of the trains.

This paper is structured as follows. Sect. 2 describes several aspects that are relevant for the allocation of running time supplements. In Sect. 3 we describe the above mentioned first stochastic optimization model. In Sect. 4 we prove that, if the train runs over just two consecutive trips and if there is a finite probability distribution of the disturbances, then the results of the stochastic optimization model converge to the true optimum if the number of realizations