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Inflationary Perturbations: The Cosmological Schwinger Effect

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Abstract. This pedagogical review aims at presenting the fundamental aspects of the theory of inflationary cosmological perturbations of quantum-mechanical origin. The analogy with the well-known Schwinger effect is discussed in detail and a systematic comparison of the two physical phenomena is carried out. In particular, it is demonstrated that the two underlying formalisms differ only up to an irrelevant canonical transformation. Hence, the basic physical mechanisms at play are similar in both cases and can be reduced to the quantization of a parametric oscillator leading to particle creation due to the interaction with a classical source: pair production in vacuum is therefore equivalent to the appearance of a growing mode for the cosmological fluctuations. The only difference lies in the nature of the source: an electric field in the case of the Schwinger effect and the gravitational field in the case of inflationary perturbations. Although, in the laboratory, it is notoriously difficult to produce an electric field such that pairs extracted from the vacuum can be detected, the gravitational field in the early universe can be strong enough to lead to observable effects that ultimately reveal themselves as temperature fluctuations in the cosmic microwave background. Finally, the question of how quantum cosmological perturbations can be considered as classical is discussed at the end of this chapter.

6.1 Introduction

The scenario of inflation was invented in order to solve puzzling issues associated with the standard hot Big Bang theory [1, 2]. Soon after its advent, it was realized that this scenario also contains a remarkable extra bonus: it gives a well-motivated mechanism for structure formation that leads to a nearly scale-invariant power spectrum [3, 4], namely exactly what is needed in order to account for various astrophysical observations in a satisfactory way [5]. However, this is not the only aspect that deserves to be stressed. Indeed, even from a fundamental point of view, this mechanism appears quite remarkable in the sense that it combines general relativity with quantum mechanics. The

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main purpose of this chapter is to thoroughly discuss this aspect of the theory of inflationary cosmological perturbations.

This theory is in fact remarkable at two levels. Firstly, because it relies on the phenomenon of particle creation which is a non-trivial effect in quantum field theory. In this sense, it is equivalent to the well-known Schwinger [6] effect and this analogy will be made explicit in this paper. The basic ingredient is a quantum scalar field Φ (in practice this is rather a fermionic field Ψ but, for simplicity, we will restrict ourselves to the case of a scalar field) interacting with a classical source, in the case of the Schwinger effect, an electric field E. The Schwinger effect has not yet been observed in the laboratory as it is difficult to produce an electric field with the required strength but there are prospects to do so, in particular at DESY with a free electron laser (FEL) in the X-ray band [7, 8, 9] but also at SLAC with the Linac Coherent Light Source (LCLS) [10]. Even if there is absolutely no reason to doubt the reality of the Schwinger effect, observing pair creation in the laboratory would clearly be a breakthrough and, in some sense, a verification of the corresponding inflationary mechanism.

Secondly, the theory of cosmological perturbations is also remarkable for the following reason. In cosmology, what plays the role of the constant electric field E [originating from a time-dependent potential vector $A_\mu(t)$] is the background gravitational field, i.e., the Friedmann–Lemaître–Robertson–Walker (FLRW) scale factor $a(t)$, and what plays the role of the quantum fermionic field $\Psi(t, \boldsymbol{x})$ is the quantum perturbed metric $\delta g_{\mu\nu}(t, \boldsymbol{x})$, that is to say the small inhomogeneous fluctuations of the gravitational field itself [11]. In the early Universe, the gravitational field is quite strong, i.e., for instance $H/m_{\rm Pl} \sim 10^{-5}$, where H is the Hubble parameter, and this is why the cosmological version of the Schwinger effect can be efficient. From the previous considerations, it is also clear that, in some sense, the inflationary mechanism relies on quantum gravity which adds another interesting aspect to the problem. Of course, we only deal with linearized quantum gravity and this is why we do not have to face tricky questions associated with finiteness of quantum gravity and/or renormalization. More precisely, in the case of scalar perturbations, $\delta g_{\mu\nu}(t, \boldsymbol{x})$ is replaced by the Mukhanov–Sasaki variable $v(t, \boldsymbol{x})$ which is a combination of the Bardeen potential (the generalization of the Newtonian potential in general relativity) and of the small fluctuations in the inflaton field. For gravitational waves, the relevant quantity is $h_{ij}(t, \boldsymbol{x})$, the transverse and traceless part of the perturbed metric.

Let us notice that the two above-mentioned aspects are features of the theory of cosmological perturbations in general. The inflationary aspect is in fact not necessary in order to have particles creation: only a dynamical background is required. However, a quasi-exponential expansion is mandatory if one wants to obtain a power spectrum which is close to scale invariance as indicated by astrophysical observations.

The fact that the inflationary mechanism for structure formation relies on general relativity and quantum mechanics also raises fundamental