Strangeness contributions to nucleon form factors

R.D. Young

Jefferson Lab, 12000 Jefferson Ave., Newport News, VA 23606, USA

Paper reprinted from Eur. Phys. J. A with permission
© Società Italiana di Fisica / Springer-Verlag 2007

Abstract. We review a recent theoretical determination of the strange quark content of the electromagnetic form factors of the nucleon. These are compared with a global analysis of current experimental measurements in parity-violating electron scattering.

PACS. 11.30.Er Charge conjugation, parity, time reversal, and other discrete symmetries – 14.20.Dh Protons and neutrons

1 Introduction

The determination of the strange quark content of the nucleon offers a unique probe to measure the nonperturbative structure of the nucleon. As the nucleon carries zero net strangeness, the influence of strange quarks arises entirely through interaction with the vacuum. Technically speaking, strange quarks directly probe the role of the fermion determinant in QCD. While strangeness measurements in nucleon structure have been difficult to isolate, the contribution of the neutral weak current in elastic scattering offers perhaps the most direct measurement of the strange quark content of the nucleon [1].

Here we review recent progress in the study of the strange quark contributions to the nucleon form factors. In sect. 2 we discuss the theoretical developments in the chiral extrapolation of lattice simulation results that have enabled a precise determination of the strangeness form factors. An outline of this determination is provided in sect. 3. In sect. 4 this theoretical prediction is compared with a global analysis of the experimental measurements searching for strangeness in the nucleon.

2 Chiral applications in lattice QCD

The computational expense of incorporating the effects of the fermion determinant has restricted modern lattice QCD simulations to the use of pion masses that are typically \( m_\pi \gtrsim 500 \text{ MeV} \). Recent progress has seen nucleon 3-point functions simulated with pion masses pushing down to the 350 MeV range [2–4], yet a reliable extrapolation in the pion mass is still required to compare with reality — until the physical point is readily accessible.

Ultimately, chiral perturbation theory offers the potential to deliver model-independent quark mass extrapolations of lattice results. As disappointing as it may be, there is mounting evidence that applications of low-order chiral expansions should be taken with serious caution beyond pion masses of the order 300 MeV [5,6]. Further, the situation could be significantly worse for observables which are particularly singular near the chiral limit, such as magnetic moments [7], charge radii or polarisabilities.

In the future, chiral extrapolations will be constrained model independently by precision, large-volume lattice calculations in the chiral regime. Until then, one requires methods which can reliably extrapolate from the moderately heavy quark mass regime, while maintaining all the constraints of the effective field theory. The best available solution is to reformulate the effective field theory using finite-range regularisation (FRR) [5].

In extrapolating lattice simulation results from beyond the chiral regime, one cannot guarantee that results will be independent of regularisation scheme. By choosing a particular scheme, one has necessarily introduced a model — whether it be FRR, or a more traditional regularisation. The advantages of FRR have been quantitatively demonstrated for the nucleon mass. Using lattice results over the range \( 0.25 \gtrsim m_\pi^2 \gtrsim 1.0 \text{ GeV}^2 \), the FRR extrapolated nucleon mass at the physical point displays less than 1% variation associated with the truncation between successive orders in the chiral expansion. Further, the sensitivity to the choice of the functional form of FRR is also less than 1% [8]. Until sufficient lattice results are available in the chiral regime, when the choice of regularisation becomes superfluous, FRR offers independent-of-model chiral extrapolations.

Because of the cost of simulating the fermion determinant, historically it has been common in lattice QCD to ignore this contribution to the path integral. This is the quenched “approximation”, where the influence...
of quark-antiquark pair creation in the vacuum is neglected. Fortunately, the study of the chiral extrapolation of baryon masses in quenched and dynamical simulations has revealed a remarkable phenomenological relation between these simulations. The differences between quenched and dynamical baryon masses are well described by the differences in the Goldstone boson loop corrections of the low-energy effective field, when evaluated with an appropriate finite-range regulator [9]. Although this is not a field-theoretic connection, the numerical success does mean that one has substantial confidence in obtaining physical estimates from quenched lattice results.

Beyond the baryon masses, the technique of chiral unquenching has been extended to the nucleon magnetic moments [7]. Here it was predicted that there should be very little difference in the quenched and dynamical nucleon magnetic moments over a large range of quark masses, with significant differences only anticipated near the chiral limit. These findings have been recently supported by first calculations with 2-flavour dynamical lattices [3, 4].

With the success of chiral extrapolations and the estimation of the effects of the quark determinant, we look to the extraction of the strangeness contributions to the nucleon electromagnetic form factors.

3 Strangeness calculation

Direct lattice QCD calculations of the strangeness content of the nucleon have been unable to produce a conclusive determination [10]. It is hoped that the next generation of calculations could shed light on this elusive signal. This may require further development of emerging lattice techniques. One potential gain could be seen by utilising background field methods [11], where a weak signal could be enhanced by coupling a strong electromagnetic field to the vacuum strange quarks. The method to evaluate the enhanced by coupling a strong electromagnetic field to the vacuum strange quarks. The method to evaluate the vacuum strange quarks. The method to evaluate the vacuum strange quarks. The method to evaluate the vacuum strange quarks. The method to evaluate the vacuum strange quarks. The method to evaluate the vacuum strange quarks. The method to evaluate the vacuum strange quarks. The method to evaluate the vacuum strange quarks. The method to evaluate the vacuum strange quarks. The method to evaluate the vacuum strange quarks. The method to evaluate the vacuum strange quarks. The method to evaluate the vacuum strange quarks. The method to evaluate the vacuum strange quarks. The method to evaluate the vacuum strange quarks. The method to evaluate the vacuum strange quarks. The method to evaluate the vacuum strange quarks. The method to evaluate the vacuum strange quarks. The method to evaluate the vacuum strange quarks. The method to evaluate the vacuum strange quarks.

Beyond the baryon masses, the technique of chiral unquenching has been extended to the nucleon magnetic moments [7]. Here it was predicted that there should be very little difference in the quenched and dynamical nucleon magnetic moments over a large range of quark masses, with significant differences only anticipated near the chiral limit. These findings have been recently supported by first calculations with 2-flavour dynamical lattices [3, 4].

With the success of chiral extrapolations and the estimation of the effects of the quark determinant, we look to the extraction of the strangeness contributions to the nucleon electromagnetic form factors.

3 Strangeness calculation

Direct lattice QCD calculations of the strangeness content of the nucleon have been unable to produce a conclusive determination [10]. It is hoped that the next generation of calculations could shed light on this elusive signal. This may require further development of emerging lattice techniques. One potential gain could be seen by utilising background field methods [11], where a weak signal could be enhanced by coupling a strong electromagnetic field to the vacuum strange quarks. The method to evaluate the enhanced by coupling a strong electromagnetic field to the vacuum strange quarks. The method to evaluate the enhanced by coupling a strong electromagnetic field to the vacuum strange quarks. The method to evaluate the enhanced by coupling a strong electromagnetic field to the vacuum strange quarks. The method to evaluate the enhanced by coupling a strong electromagnetic field to the vacuum strange quarks. The method to evaluate the enhanced by coupling a strong electromagnetic field to the vacuum strange quarks. The method to evaluate the enhanced by coupling a strong electromagnetic field to the vacuum strange quarks. The method to evaluate the enhanced by coupling a strong electromagnetic field to the vacuum strange quarks. The method to evaluate the enhanced by coupling a strong electromagnetic field to the vacuum strange quarks. The method to evaluate the enhanced by coupling a strong electromagnetic field to the vacuum strange quarks. The method to evaluate the enhanced by coupling a strong electromagnetic field to the vacuum strange quarks. The method to evaluate the enhanced by coupling a strong electromagnetic field to the vacuum strange quarks. The method to evaluate the enhanced by coupling a strong electromagnetic field to the vacuum strange quarks. The method to evaluate the enhanced by coupling a strong electromagnetic field to the vacuum strange quarks.

The strangeness magnetic moment can be written as

\[
G_M^s = \frac{1}{1 - \frac{R_d}{R_u}} \left[ 2p + n - \frac{w^p}{w^s} (\Sigma^+ - \Sigma^-) \right],
\]

\[
G_M^\lambda = \frac{1}{1 - \frac{R_d}{R_u}} \left[ p + 2n - \frac{w^n}{w^\Lambda} (\Xi^0 - \Xi^-) \right],
\]

where \( p, n, \Sigma^\pm \) and \( \Xi^{0/-} \) denote the experimentally measured magnetic moments of the respective baryon. The formulae rely on two inputs from lattice simulations.

The first is the ratio \( w^p/w^\Sigma \), which measures the relative strength of the valence (fig. 1, left) \( u \)-quark contribution in the proton relative to the \( \Sigma^+ \) —or similarly \( w^n/w^\Xi \) in eq. (2). The second is \( R_d^u \), which describes the ratio of the strange-to-light disconnected (fig. 1, right) contributions. Equating eqs. (1) and (2) and using the experimental magnetic moments, produces a linear relationship between the two unknown valence ratios. This constraint, a result of charge symmetry alone, is displayed in fig. 2. The line is divided by two segments, where the sign of \( G_M^s \) can be determined under the quite general assumption that \( 0 > R_d^u \geq 1 \). Recently, it has been suggested that there could be a sign change in this ratio between the heavy-quark limit and naive expectations in the Goldstone boson sector [17].

Given that the properties of the kaon are much more Goldstone-like than a heavy-light meson, and that the heavy-quark limit of \( \mu_u/\mu_n \) is approached very slowly [18], it should not be expected that the strange quark could be reliably described by heavy-quark effective theory.

The techniques discussed in sect. 2 were applied to determine the ratios \( w^p/w^\Sigma \) and \( w^n/w^\Xi \), appearing in eqs. (1) and (2). The analysis has utilised a high-precision numerical study of the baryon electromagnetic form factors in quenched lattice QCD [19]. Upon performing finite-volume corrections, adjustments for the quenched approximation and a controlled chiral extrapolation, the resulting