A Selective Approach to Conformal Refinement of Unstructured Hexahedral Finite Element Meshes

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Summary. Hexahedral refinement increases the density of an all-hexahedral mesh in a specified region, improving numerical accuracy. Previous research using solely sheet refinement theory made the implementation computationally expensive and unable to effectively handle concave refinement regions and self-intersecting hex sheets. The Selective Approach method is a new procedure that combines two diverse methodologies to create an efficient and robust algorithm able to handle the above stated problems. These two refinement methods are: 1) element by element refinement and 2) directional refinement. In element by element refinement, the three inherent directions of a Hex are refined in one step using one of seven templates. Because of its computational superiority over directional refinement, but its inability to handle concavities, element by element refinement is used in all areas of the specified region except regions local to concavities. The directional refinement scheme refines the three inherent directions of a hexahedron separately on a hex by hex basis. This differs from sheet refinement which refines hexahedra using hex sheets. Directional refinement is able to correctly handle concave refinement regions. A ranking system and propagation scheme allow directional refinement to work within the confines of the Selective Approach Algorithm.

1 Introduction

As computing power continues to increase, the finite element method has become an increasingly important tool for many scientists and engineers. An essential step in the finite element method involves meshing or subdividing the domain into a discrete number of elements. Mesh generation has therefore been the topic of much research. Tetrahedral (Tet) or hexahedral (Hex) elements are commonly used to model three dimensional problems. Tet elements have extremely robust modeling capabilities for any general shape while Hex elements provide more efficiency and accuracy in the computational process [1].
Within the realm of hexahedral mesh generation, mesh modification is an area of research that attempts to improve the accuracy of an analysis by locally modifying the mesh to more accurately model the physics of a problem. Hexahedral refinement modifies the mesh by increasing the element density in a localized region.

Several schemes have been developed for the refinement of hexahedral meshes. Methods using iterative octrees\cite{2} have been proposed, however these methods result in nonconformal elements which cannot be accommodated by some solvers. Other techniques insert non-hex elements that result in hybrid meshes or require uniform dicing to maintain a consistent element type\cite{3}. Schneiders proposed an element by element refinement scheme\cite{4} in connection with an octree-based mesh generator, however this technique is limited in that it is unable to handle concavities (see Section 2.2). Schneiders later proposed a sheet refinement method\cite{5} which produces a conformal mesh by pillowing layers in alternating i, j, and k directions but relies on a Cartesian initial octree mesh. Tchon et al. built upon Schneiders’ sheet refinement in their 3D anisotropic refinement scheme by expanding the refinement capabilities to unstructured meshes\cite{6}\cite{7} however this scheme still has poor scalability inherent in all sheet refinement schemes. Harris et al. further expanded upon Schneiders’ and Tchon’s work by using templates instead of pillowing to refine the mesh and included capabilities to refine element nodes, element edges, and element faces\cite{8}. While the refinement scheme introduced by Harris is robust in many aspects, it is limited by self-intersecting hex sheets (see Section 2.2), concavities, and poor scalability. The refinement process developed in this paper combines the element by element method proposed by Schneiders and the sheet refinement method proposed by Harris to create a method that overcomes the limitations of using either method alone.

2 Background

A hexahedron, the finite element of interest in this paper, has a dual representation defined by the intersection of three sheets called twist planes\cite{9}\cite{10}. Each sheet represents a unique and inherent direction within a hexahedron. Figure 1 shows a hexahedron with its three dual twist planes. Each plane represents a unique direction of refinement.

2.1 Element by Element Refinement

Element by element refinement replaces a single hexahedron with a predefined group of conformal elements effectively refining all three directions of the hexahedron at the same time. As such a nonconformal mesh is temporarily created until all templates have been inserted. Only one template is applied to any initial element thus increasing the efficiency of the refinement process. Figure 2 shows how a mesh is refined using element by element refinement.