Reachability and Dependency Calculi: Reasoning in Network Algebras

Alexander Scivos

Institut für Informatik,
Albert-Ludwigs-Universität Freiburg
Georges-Köhler-Allee 52,
79110 Freiburg, Germany
scivos@informatik.uni-freiburg.de

Abstract. Reasoning in complex systems of dependencies is important in our highly connected world, e.g. for logistics planning, and for the analysis of communication schemes and social networks. Directed graphs are often used to describe scenarios with links or dependencies. However, they do not reflect uncertainties. Further, hardly any formal method for reasoning about such systems is in use. As it is hard to quantify dependencies, calculi for qualitative reasoning (QR) are a natural choice to fill this gap. However, QR is so far concentrated on spatial and temporal issues. A first approach is the dependency calculus $DC$ for causal relations [15], but it cannot describe situations in which cycles might occur within a graph. In this paper, refinements of $DC$ meeting all requirements to describe dependencies on networks are investigated with respect to satisfiability problems, construction problems, and tractable subclasses.

1 Introduction

Reasoning about complex dependencies between events is a crucial task in many applications of our highly partitioned, but widely linked world. Whenever the required answer is a decision or classification, Qualitative Reasoning (QR) is best-suited: It abstracts from metrical details of the physical world and enables computers to make predictions about relations, even when precise quantitative information is not available or irrelevant [3]. QR is an abstraction that summarizes similar quantitative states into one qualitative characterization. From the cognitive perspective, the qualitative method categorizes features within the object domain rather than by measuring them in terms of some external scale [7]. This is the reason why qualitative descriptions are quite natural for humans.

The two main directions in QR so far are spatial and temporal reasoning. Topics of spatial reasoning comprise topological reasoning about regions [16], positional reasoning about point configurations [17], and reasoning about directions [49]. For temporal reasoning, either points or intervals are used as basic entities [19].

In contrast, this paper elaborates a new direction in QR: reasoning about links and dependencies. For describing linked systems, directed graphs (networks) are established. Network links can represent links in the internet, cash flow, railway connections, road...
systems, spreading of diseases (for medical analysis), information passed on in an organization, and genetic inheritance. There are many algorithms known for answering specific questions about a given network, e.g. if it is connected, or for finding the shortest path between two entities in it [5]. However, these algorithms assume that the network is defined, i.e. for each pair of entities, it is known how they are linked directly. For finding a network with specific properties and for reasoning with uncertainties, however, no calculus has been established yet.

A network of warehouses with possible delivery routes might illustrate such questions (cf. Fig. 1b). If a network is given, e.g. the following questions can be asked: Can site e deliver to a? Is there a route through f that runs in a cycle? If flawed goods are delivered to site c and to site d, is it possible that they come from a common source?

Fig. 1. Examples for networks: Train connections, delivery routes, abstract directed graph

Now assume that only for some pairs, we impose constraints on the relationship between them, but for other pairs, we are undecided: They might be linked or not. For example, is it possible to change the connection between e and g leaving all others as they are so that there is no cycle in the entire system? Or, can a link between e and f be inserted in either direction that is not part of a cyclic route?

Consider a more abstract level of reasoning: Without any other constraints, is it possible to construct a network in which a and b can deliver to each other, and c can be reached from a, but not from b? What follows for the relationship between a and b if d is directly linked to a, and if there is some cyclic path through b and d? This paper aims at giving a formal calculus that answers such questions automatically.

Different kinds of properties will be expressed by different labels.

1.1 Related Work

Of all spatial calculi, the point algebras for linear and partial orders come closest to the network calculi. The linear point algebra $PA_{lin}$ was introduced 1989 by Vilain [19] with the basic labels $\{<, =, >\}$. The general satisfiability problem for $PA_{lin}$ is in $\text{PTIME}$ [19]. However, real-world problems do not necessarily have linear structures as underlying space. In the nineties, the development of extensions of the linear case into nonlinear structures started to address such problems. Broxvall [2] showed that the constraint satisfaction problem for the point algebra on nonlinear structures $PA_{po}$ with the basic labels $\{<, =, >, ||\}$ is $\text{NP}$-hard. $\text{NP}$-hard problems usually have interesting fragments, the so-called tractable subclasses. A fragment of a relational algebra is a subset of relations that is closed under composition and converse. A subclass is called tractable if satisfiability can be decided in polynomial time. Broxvall identified three maximal tractable fragments of $PA_{po}$. Normally, for these classes the path-consistency method [12] decides satisfiability.