Modeling Flow Transition in Hypersonic Boundary Layer

S. Fu1*, L. Wang1

1School of Aerospace Engineering, Tsinghua University, Beijing 100084, China
Email: wangliang99@mails.tsinghua.edu.cn

Abstract A new transition/turbulence model considering the modes of instability and the compressibility effects is established. Based on the Menter’s SST model, it adds the contribution of nonturbulent fluctuations into the effective viscosity. The present model is validated by comparing the results of subsonic flat plate, supersonic straight cone and hypersonic flared cones at zero angle of attack with available experiments. In general, good agreement is indicated.

Key words: hypersonic boundary-layer, transition, turbulence model, intermittency factor

INTRODUCTION

Boundary layer transition is an important issue in the design of hypersonic atmospheric and reentry vehicles, especially for the thermal protection and propulsion systems, because the heat transfer and skin friction increase rapidly or are maximum within the transitional region. Delaying the transition can thus result in significant reduction of the aerodynamic drag and heating loads. Empirical methods correlate the transition Reynolds number (based on momentum thickness) with freestream and boundary conditions. No details of the disturbance mechanisms that cause the transition are included. The stability theory describes the behavior of the disturbances in the flow, providing deeper physical understanding of the transition process. Application of DNS in hypersonic boundary layer is far from practical. The RANS approach is still the main tool in the transition/turbulence modeling in the current CFD although the existing transition model badly requires further improvement. The purpose of this investigation is to develop an improved flow transition model for hypersonic boundary-layer transition with the aid of flow stability theory.

Many researchers employ low-Reynolds number turbulence models (some with special modifications) to simulate the transition process. As TRANSPRE TURB European project realized that turbulence model without making use of the intermittency proves to be very delicate and often extremely unreliable in the prediction of transition. So there appear lots of models involving intermittency factor, some build the transport equation for it and obtain success in simulations of the transition on flat plate and airfoil in low-speed flow. A recent example is the work by Suzen and Huang [1]. However, these models include non-local formulations. For general purpose CFD applications, Langtry and Menter develop a transition model based strictly on local variables, which is now implemented in the software package CFX-5 [2]. The work by Walters and Leylek has similar characters [3].

PROBLEM FORMULATION

Current models involving intermittency factor are not appropriate for the transition in supersonic flows or the cross-flow transition. One reason is that they do not account for the non-turbulent fluctuations that eventually lead to transition. For the eddy-viscosity models, the viscosity in Navier-Stokes code is usually set as

\[ \mu = \mu_t + \mu_{eff} \]  

(1)

where \( \mu_t \) and \( \mu_{eff} \) represent molecular viscosity and effective viscosity, respectively. According to Simon and Stephens’ assumption [4], those models all treat \( \mu_{eff} \) as the multiplication of the intermit-
tency factor $\gamma$ and the eddy viscosity $\mu_t$, i.e. $\gamma \mu_t$. In a different approach, Warren and Hassan [5] give their proposal as

$$\mu_{eff} = (1 - \gamma)\mu_{nt} + \gamma \mu_t$$

where the subscript $nt$ denotes to the contribution of the non-turbulent fluctuations. This approach seems more reasonable thus is also used in the present work. Furthermore, without invoking the nature of non-turbulent fluctuations, their constitutive stress-strain relations are assumed to be similar with the ones for turbulent fluctuations, thus $\mu_{nt}$ is expressed as

$$\mu_{nt} = C_{\mu} \rho k \tau_{nt}, C_{\mu} = 0.09$$

where $k$ is the fluctuation kinetic energy per unit mass, $\rho$ is the density and $\tau_{nt}$ is a characteristic timescale of the type of instabilities being considered. Therefore, the model focuses on the determination of $\tau_{nt}$ which is derived from the linear stability theory (LST).

When the edge Mach number $Ma_e < 4$, the boundary layer transition is dominated by first mode disturbances, which is called Tollmien-Schlichting waves in two-dimensional incompressible flows. For this case the characteristic timescale is set as

$$\tau_{nt1} = a/\omega_1$$

where $a = 0.069(Tu - 0.138)^2 + 0.00819$, $Tu$ is the freestream intensity (FSTI); $\omega_1$ is the maximum-amplified frequency of the first mode disturbance given by Walker [6]. If $Ma_e > 4$, second mode is the dominant mode of instability, which is the most unstable of Mack modes. For this mode the timescale is given by

$$\tau_{nt2} = b/\omega_2, \quad \omega_2 = U_p/\lambda$$

where $b$ is also a model constant, the specified phase velocity $U_p$ and wavelength $\lambda$ are approximately the edge velocity and two times the boundary layer thickness, respectively. Therefore the total transitional contribution to the viscosity timescale $\tau_{nt}$ is set as the sum of $\tau_{nt1}$ and $\tau_{nt2}$.

Similarly, the decay time $\tau_k$ for the turbulent kinetic energy is modeled using contribution from non-turbulent and turbulent fluctuation as

$$\tau_k = 1/[(1 - \gamma)/\tau_{k,nt} + \gamma/\tau_{k,t}], \quad \tau_{k,nt} = (a + b)(\mu_t/\mu)S, \quad \tau_{k,t} = 1/\beta \omega$$

where $S$ is the magnitude of the mean strain rate and $\omega$ is the specific dissipation rate of $k$. Eqs. (2) and (6) are then combined with SST $k-\omega$ model [7]

$$\frac{\partial(\rho k)}{\partial t} + \frac{\partial(\rho u_j k)}{\partial x_j} = P_k - \frac{\rho k}{\tau_k} + \frac{\partial}{\partial x_j} \left\{ \left( \mu + \sigma_k \mu_{eff} \right) \frac{\partial k}{\partial x_j} \right\}$$

$$\frac{\partial(\rho \omega)}{\partial t} + \frac{\partial(\rho u_j \omega)}{\partial x_j} = P_\omega - D_\omega + C_d \omega + \frac{\partial}{\partial x_j} \left\{ \left( \mu + \sigma_\omega \mu_{eff} \right) \frac{\partial \omega}{\partial x_j} \right\}$$

Eqs. (7) would degenerate into the standard SST mode at $\gamma = 1$, indicating that $\gamma$ plays as the weight number between the non-turbulent and the turbulent flow. $\gamma$ is calculated by the correlation of Dhawan and Narasimha [8] with the corresponding transition onset position well represented by Warren and Hassan’s criteria [5].

The present calculation also finds necessary to adapt some compressibility corrections in the turbulence transport equations to account for the dilatation-dissipation, the pressure-dilatation and mass flux terms.

RESULTS AND DISCUSSION

The present model proposal is calibrated and validated with three sets of experimental data involving subsonic flow over a flat plate, supersonic flow over a straight cone and hypersonic flow over a flared cone at zero angle of attack. The last two data sets come from NASA Langley ‘Quiet Nozzle’. The Mach numbers are 0.147, 3.5 and 5.91, respectively.