23 Fuzzy Transportation Problem

23.1 Introduction

In this chapter we apply our fuzzy Monte Carlo method to determine approximate solutions to a fuzzy transportation problem. The next section presents the standard definition of the transportation model. Then in Section 23.3 we fuzzify it and apply our fuzzy Monte Carlo method to generate approximate solutions. An example is given which is to show the results of our fuzzy Monte Carlo method.

23.2 Transportation Problem

The standard transportation model seeks to find a transportation plan for a single commodity from a number of sources to a number of destinations [8]. The data in the model includes: (1) the amount of supply at each source and the demand at each destination; and (2) the unit transportation cost of the commodity from each source to each destination. A destination may receive its demand from many sources. The objective is to determine the shipping plan, to meet all demands but not exceed any supply, to minimize the total transportation cost.

Assume there are \( m \) sources and \( n \) destinations. Let \( x_{ij} \) be the amount to be shipped from source \( i \) to destination \( j \), \( 1 \leq i \leq m \), \( 1 \leq j \leq n \). \( x_{ij} \) will be an integer greater than, or equal to, zero. Also let \( a_i \) be the amount of supply at source \( i = 1, \ldots, m \) and \( b_j \) the demand at destination \( j = 1, \ldots, n \). The unit transportation cost from source \( i \) to destination \( j \) is \( c_{ij} \), \( 1 \leq i \leq m \), \( 1 \leq j \leq n \).

The linear programming model is

\[
\text{minimize } Z = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij}, \quad (23.1)
\]

subject to

\[
\sum_{i=1}^{m} x_{ij} \geq b_j, \quad j = 1, \ldots, n, \quad (23.2)
\]
and
\[ \sum_{j=1}^{n} x_{ij} \leq a_i, \; i = 1, \ldots, m, \tag{23.3} \]
and \( x_{ij} \geq 0 \) and \( x_{ij} \) integer. For the model to be feasible we must have the total supply is at least equal to the total demand
\[ \sum_{i=1}^{m} a_i \geq \sum_{j=1}^{n} b_j. \tag{23.4} \]

This problem has a special solution algorithm and is not usually solved as a linear program [8].

### 23.3 Fuzzy Transportation Problem

Now we allow some, or all, of the parameters \( c_{ij}, a_i \) and \( b_j \) to be fuzzy showing any uncertainty in their values. However, the \( x_{ij} \) will be crisp because if we allowed them to be fuzzy we would have to defuzzify them at the end to obtain a feasible shipping plan. Let \( \overline{c}_{ij} \) be a triangular fuzzy number representing the cost of sending one unit from source \( i \) to destination \( j \). \( \overline{A}_i \) is a triangular fuzzy number for the amount at source \( i \) and \( \overline{B}_j \) is another triangular fuzzy number for the demand at destination \( j \). Some of these parameters may be crisp. Then the fuzzy optimization problem to solve is
\[
\text{minimize } Z = \sum_{i=1}^{m} \sum_{j=1}^{n} \overline{c}_{ij} x_{ij},
\tag{23.5}
\]
subject to
\[ \sum_{i=1}^{m} x_{ij} \geq \overline{B}_j, \; j = 1, \ldots, n, \tag{23.6} \]
and
\[ \sum_{j=1}^{n} x_{ij} \leq \overline{A}_i, \; i = 1, \ldots, m, \tag{23.7} \]
and \( x_{ij} \geq 0 \) and \( x_{ij} \) integer. For the model to be feasible we must have the total supply is at least equal to the total demand
\[ \sum_{i=1}^{m} \overline{A}_i \geq \sum_{j=1}^{n} \overline{B}_j. \tag{23.8} \]

There have been numerous papers on the fuzzy transportation problem (see [1]-[7]). These authors allow for fuzzy demand and fuzzy supply and/or fuzzy cost coefficients. However, they keep the flow from source \( i \) to destination \( j \) (the \( x_{ij} \)) non-negative integers. We will do the same but apply our fuzzy Monte Carlo method to generate (approximate) solutions.