Modeling Synchronized Flow at Highway Bottlenecks

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Summary. Many experimental studies have shown the appearance of synchronized flow at highway bottlenecks. We study highway bottlenecks within the macroscopic BVT model. The BVT model describes traffic flow as a hyperbolic system of balance laws. It generalizes the traffic model of Aw, Rascle and Greenberg by introducing in the momentum equation a new source term, which can become negative due to the finite reaction and relaxation times of drivers. The model is capable of reproducing multivalued fundamental diagrams, the metastability of free traffic flow at the onset of instabilities and wide moving jams. Based on previous work we describe the coupling conditions for the Riemann problem of the system and apply them to highway bottlenecks. We focus our study on the situation where the bottlenecks are either caused by the reduction of the number of lanes or by on-ramps or off-ramps. Our numerical simulations reproduce the appearance of synchronized flow at these highway bottlenecks. The analysis of the lane reduction setup shows that the outflow from the synchronized flow region in front of the bottleneck is constant and below the maximum free flow. This observation can be understood from the study of the static solutions within the model. As a consequence of the coupling conditions static solutions have to cross the jam line, one of the additional equilibrium solutions within the BVT model. This crossing determines the flow value of the static solution.

1 Balanced Vehicular Traffic: The BVT Model

The BVT model (balanced vehicular traffic model, see [1, 2]) generalizes the traffic model of Aw, Rascle and Greenberg (often called Aw, Rascle and Zhang model, too, see [3–5]) by prescribing a more general source term to the pseudo-momentum equation. The evolution of traffic density $\rho$ and dynamical velocity $v$ is described by the following hyperbolic system of balance laws
∂ρ
∂t + ∂(ρv)
∂x = 0,

(1a)

∂(ρ(v − u(ρ)))
∂t + ∂(ρv(v − u(ρ)))
∂x = β(ρ, v)ρ(u(ρ) − v),

(1b)

where \( t \) denotes the time coordinate and \( x \) the space coordinate. The function \( u(\rho) \) denotes the equilibrium velocity, which is monotonously decreasing, \( β(\rho, v) \) is the effective relaxation coefficient. Due to finite reaction times and finite relaxation times, an effective relaxation coefficient results with negative values in the neighborhood of the equilibrium flow curve \( u(\rho) \) for medium to high traffic densities \( \rho \) [1]. As a consequence of the negative relaxation coefficients, there are two additional equilibrium velocity curves, the so-called high-flow branch and the jam line. The high-flow branch is metastable for medium traffic densities and unstable for high traffic densities, whereas the jam line is unstable for medium traffic densities and metastable for high traffic densities, see [2, 6] for a detailed discussion. We stress, that the BVT model, like the model of Aw, Rascle and Greenberg, fulfills the anisotropic condition, the characteristic speeds of the system \( \lambda_1 = v − \rho u'(\rho) \) and \( \lambda_2 = v \) are bounded from above by the vehicle speed \( v \). Hence the model does not show the unphysical behavior pointed out by Daganzo [7]. Moreover, the subcharacteristic condition [8] is essential for the stability properties above and can explain the form of the reversed \( \lambda \) observed in the fundamental diagram of traffic flow [9].

Here we are interested in synchronized flow at highway bottlenecks [10]. We applied the coupling conditions for the Riemann problem of the system and studied highway bottlenecks either caused by the reduction of the number of lanes or by on- and off ramps [11]. Our results show that the BVT model can explain the observation of a capacity drop at highway bottlenecks, i.e. the outflow from the bottleneck regions is substantially lower than the maximum capacity of free flow (see e.g. [12, 13]).

2 Coupling Conditions at Intersections

In order to model bottlenecks in the BVT model one has to describe the coupling conditions at intersections or junctions. These coupling conditions give rise to the boundary values for vehicle and pseudo-momentum fluxes of each road section, which are necessary to solve the corresponding initial value problem in each section.

Although the source term in the pseudo-momentum equation, i.e. the right-hand side of Eqn. (1b), plays an essential role for the traffic dynamics on road sections, it can be neglected for the analysis of the Riemann problem at intersections, since it is never a delta-function. For the homogeneous system without this source term, i.e. the Aw-Rascle model [3], there is a large amount of theoretical work on the coupling conditions at intersections [14–18]. For comments on the difference between these approaches, see [11].