A Unified Approach to Congestion Games and Two-Sided Markets

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Abstract. Congestion games are a well-studied model for resource sharing among uncoordinated selfish agents. Usually, one assumes that the resources in a congestion game do not have any preferences over the players that can allocate them. In typical load balancing applications, however, different jobs can have different priorities, and jobs with higher priorities get, for example, larger shares of the processor time. We introduce a model in which each resource can assign priorities to the players and players with higher priorities can displace players with lower priorities. Our model does not only extend standard congestion games, but it can also be seen as a model of two-sided markets with ties. We prove that singleton congestion games with priorities are potential games, and we show that every player-specific singleton congestion game with priorities possesses a pure Nash equilibrium that can be found in polynomial time. Finally, we extend our results to matroid congestion games, in which the strategy space of each player consists of the bases of a matroid over the resources.

1 Introduction

In a congestion game, there is a set of players who compete for a set of resources. Each player has to select a subset of resources that she wishes to allocate. The delay of a resource depends on the number of players allocating that resource, and every player is interested in allocating a subset of resources with small total delay. Congestion games are a well-studied model for resource sharing among uncoordinated selfish agents. They are widely used to model routing, network design, and load balancing. One appealing property of congestion games is that they are potential games. In particular, this implies that every congestion game possesses a pure Nash equilibrium and that myopic player eventually reach a Nash equilibrium by iteratively playing better responses.

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One drawback of the standard model of congestion games is that resources do not have any preferences over the players. In typical load balancing applications, however, different jobs have different priorities, and depending on the policy, jobs with a low priority are stopped or slowed down when jobs with higher priorities are present. We introduce congestion games with priorities to model the scenario in which a job can prevent jobs with lower priorities from being processed. In our model, each resource can partition the set of players into classes of different priorities. As long as a resource is only allocated by players with the same priority, these players incur a delay depending on the congestion, as in standard congestion games. But if players with different priorities allocate a resource, only players with the highest priority incur a delay depending on the number of players with this priority, and players with lower priorities incur an infinite delay. Intuitively, they are displaced by the players with the highest priority. This model is applicable if every player controls a stream of jobs rather than only a single one. In the latter case, it might be more reasonable to assume that jobs with lower priorities incur a large but finite delay.

Motivated by the application of congestion games to load balancing, we mainly consider congestion games in which each player has to choose exactly one resource to allocate, namely one server on which her job is to be processed. Such singleton congestion games or congestion games on parallel links have been studied extensively in the literature \[4,8,9,14\]. Moreover, we show that singleton congestion games with priorities are potential games, implying that uncoordinated players who iteratively play better responses eventually reach a pure Nash equilibrium. If all resources have the same priorities, then we even obtain polynomial-time convergence to a Nash equilibrium. Milchtaich \[19\] introduces player-specific congestion games as an extended class of congestion games in which every player can have her own delay function for every resource. Milchtaich shows that player-specific singleton congestion games are not potential games anymore but that they possess pure Nash equilibria that can be computed in polynomial time. We show that also in player-specific singleton congestion games with priorities pure Nash equilibria exist that can be computed efficiently.

Interestingly, our model of player-specific congestion games with priorities does not only extend congestion games but also the well-known model of two-sided markets. This model was introduced by Gale and Shapley \[10\] to model markets on which different kinds of agents are matched to another, for example men and women, students and colleges \[10\], interns and hospitals \[22\], and firms and workers. Using the same terms as for congestion games, we say that the goal of a two-sided market is to match players and resources (or markets). In contrast to congestion games, each resource can only be matched to one player. With each pair of player and resource a payoff is associated, and players are interested in maximizing their payoffs. Hence, the payoffs implicitly define a preference list over the resources for each player. Additionally, each resource has a preference list over the players that is independent of the profits. Every player can propose to one resource and if several players propose to a resource, only the most preferred player is assigned to that resource and receives the corresponding payoff. This