

# Scheduling School Meetings

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**Abstract.** Prespecified meetings between teachers and parents have to be scheduled. All meetings have the same duration. The goal is in finding a schedule minimizing the total time and the parents' idle times. This NP-hard problem is addressed by solving first a sequence of weighted assignment problems and then performing a large scale neighborhood search based on finding negative cost cycles and shortest paths in directed graphs. This approach provides good computational results. Finally a variant of the problem with two different durations for the meetings is considered.

## 1 Introduction

We address the following problem arising in Italian high schools: on certain days of the school year, parents can meet teachers to discuss their children. Each parent tries to meet some teachers and the meetings are individual. There is no advance planning for the event and therefore parents wait in lines for a long time (one line for each teacher), not only wasting time but also preventing the possibility of meeting several teachers.

In this paper we propose a planning method in order to guarantee that each parent meets all required teachers and the global wasted time of the parents is minimized. A prerequisite for the method to work is that all meetings last the same amount of time.

The problem we study is NP-hard. We suggest a two-phase heuristic approach for its solution. The first phase computes a schedule of minimum time to allocate all meetings by solving a sequence of weighted assignment problems. The second phase minimizes the parents' idle times through a large-scale neighborhood search based on negative cycle detection and shortest path computations in directed graphs. It turns out that this search is quite effective and strongly reduces the wasted time of the final solutions.

We also briefly consider an extension of the model by allowing double duration to certain specified meetings that require more discussion.

To the best of our knowledge, this specific timetable problem has not been addressed in the literature. Nevertheless, a similar problem was considered in [3] to schedule job interviews for law firms and students at the Southeastern Public Interest Job Fair, a law fair which is held each year in the USA. In [3] the timetable problem is modeled as an edge-coloring problem on bipartite graphs

and the minimization of the idle times (considered both for law firms and students) is partially carried out by maximizing the cumulative number of meetings assigned to each period. Our problem has also some resemblance with the no-wait open shop problem [8], where it is required that all the operations of a same job are executed contiguously, i.e. no idle time for the jobs is allowed. The particular case of unit processing times has been studied in [6] and, under the additional condition of no idle time for the machines, in [5].

As pointed out above, the local search procedure we adopt is based on neighborhoods of exponential size in which an improving neighbor can be found in polynomial time by dynamic programming. The potential of using optimization methods, and in particular network flow techniques and dynamic programming, to search neighborhoods of very large size, has been remarked on in [1]. In the particular field of timetabling, large-scale neighborhood metaheuristics have been applied, for instance, in [4] to solve a nurse scheduling problem and in [9] to solve the examination timetabling problem.

The paper is organized as follows. In Section 2 we define the problem. In Section 3 an algorithm to find a schedule of minimum makespan is presented. In Sections 4 and 5 two local search procedures based on finding sequences of meeting exchanges are described. The first one considers meetings of a fixed teacher, whereas the second one considers meetings of a fixed parent. The techniques are illustrated with an example in Section 6. In Section 7 we extend the problem to the case with two different durations of meetings. Finally some computational results are given in Section 8.

## 2 Problem Definition

In the problem we consider, a set  $J$  of parents and a set  $I$  of teachers are given and each parent  $j$  wants to meet a specified subset  $I_j$  of teachers. The subsets  $I_j$  are the input data. Let  $J_i := \{j : i \in I_j\}$  be the subset of parents that want to meet the teacher  $i$ .

Assuming that all meetings last the same amount of time, called *time slot*, the output is a schedule  $t(i, j)$  assigning the time slot  $t(i, j)$  to the meeting of parent  $j$  with teacher  $i$ , with the obvious requirement that  $t(i, j) \neq t(i, j')$  for all  $j' \neq j$  and all  $i$  and also  $t(i, j) \neq t(i', j)$  for all  $i' \neq i$  and all  $j$ . We define as *makespan* of the schedule the maximum time in which a meeting occurs: i.e.,  $\max_{i \in I} t(i, j) = \max_{j \in J} t(i, j)$ . Moreover, for any given schedule, we define *idle time* for parent  $j$  any time slot  $k$  such that  $\min_i t(i, j) < k < \max_i t(i, j)$  and  $k \neq t(i, j)$  for any  $i$ . In other words, an idle time is a waiting time slot in between meetings and therefore counts as a wasted time.

We define as *school meeting problem* the problem of determining a schedule of minimum makespan that minimizes the total number of idle times over all parents. In this way the two objectives of minimizing the makespan and the wasted time of the parents are treated in a lexicographic order. The problem of minimizing the number of idle times within a fixed time is NP-hard, as can be seen by transforming the no-wait open shop problem with 0–1 processing