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Evolutionary Stochastic Portfolio Optimization

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Summary. In this chapter, the concept of evolutionary stochastic portfolio optimization is discussed. Selected theory from the fields of Stochastic Programming, evolutionary computation, portfolio optimization, as well as financial risk management is used to derive a generalized framework for computing optimal financial portfolios given an uncertain future using a probabilistic risk measure approach. A set of structurally different risk measures - Standard Deviation, Mean-absolute Downside Semi Deviation, Value-at-Risk, and Expected Shortfall - which are commonly used for practical portfolio management purposes have been selected to substantiate the approach with numerical results.

5.1 Introduction

During recent years, the increasing need for financial decision optimization algorithms for complex, and often non-convex optimization problems in the area of financial engineering led to a significant increase in the use of biologically inspired algorithms for practical financial management purposes, see e.g. (4). In this chapter, the well-known technique of Stochastic Programming is applied to solve financial portfolio optimization problems under uncertainty based on probabilistic risk measures. Evolutionary computation methods are exploited to allow for a generalization of the underlying problem structure and to solve the resulting optimization problems numerically in a systematic way. This chapter is organized as follows. The remainder of section 5.1 contains a short summary of Stochastic Programming, as well as an overview of previous evolutionary approaches to portfolio optimization. Section 5.2 surveys the field of (stochastic) portfolio optimization, and discusses the generalization of the well-known Markowitz portfolio approach to scenario-based Stochastic Programming. Furthermore, the loss-distribution based approach and its relation to probabilistic risk measures within the evolutionary optimization process is explained. Section 5.3 presents ideas and strategies for implementing a successful evolutionary portfolio optimization framework based on the discussion in section 5.2, and contains a section on different constraint handling techniques. Section 5.4 outlines details of the implementation, and presents a set of numerical results, while section 5.5 concludes this chapter.

R. Hochreiter: Evolutionary Stochastic Portfolio Optimization, Studies in Computational Intelligence (SCI) 100, 67–87 (2008)
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5.1.1 Stochastic Programming

To summarize the concept of Stochastic Programming, consider the classical deterministic optimization problem, where a decision maker aims at finding an optimal (numerical) decision $x \in \mathbb{R}^n$ by minimizing a deterministic cost function $f(\cdot)$ (or by maximizing a profit function respectively) given a set $\mathcal{X}$ of constraints, which generally consists of various physical, organizational, and regulatory restrictions. The mathematical formulation of this problem can be simplified to the formulation shown in Equ. (5.1).

\[
\begin{align*}
\text{optimize} & \quad f(x) \\
\text{subject to} & \quad x \in \mathcal{X}.
\end{align*}
\]  

During the 1950s Stochastic Programming was initiated by the seminal papers of Dantzig (6) and Beale (2). It is one of the main techniques for modeling and solving decision optimization problems under uncertainty, which is a class of optimization problems inherent to the application area of financial engineering. Due to the recent developments both from the computational and the algorithmic viewpoint, solutions of large stochastic programs are generally computable using standard computer hardware. The idea is to replace deterministic parameters by probability distributions on some probability space $(\Omega, \mathcal{F}, \mathbb{P})$, which will be denoted by $\Xi$ in the following, and to optimize a stochastic cost (or profit) function $f(\cdot, \cdot)$ over some probability functional $\mathbb{P}$. A common choice regarding this functional is the expectation $\mathbb{E}$. As Rockafellar (23) points out, expectations are only suitable for situations where the interest lies in long-range operation, and stochastic ups and downs can safely average out, which is not the case for controlling financial market risk. The recent progress of unifying probabilistic risk measures, as presented in the seminal paper by Artzner et al. (1) on coherent risk measures, motivated for using probability functionals based on such financial risk measures. See the recent book (22) for more details on modeling, measuring, and managing risk for this class of optimization applications. A different view on the integration of risk measures using the concept of deviation measures is shown in (25). In summary, the resulting mathematical meta-formulation of a stochastic program for arbitrary probability functionals is shown in eq. (5.2).

\[
\begin{align*}
\text{optimize} & \quad \mathbb{P}(f(x, \Xi)) \\
\text{subject to} & \quad (x, \Xi) \in \mathcal{X}.
\end{align*}
\]  

However, a concrete reformulation of this meta-model into some model, which can be solved with a numerical optimizer depends to a high degree on the chosen probability functional, as well as on the structure of the underlying probability space. The interested reader is referred to (26) for a recent theoretical overview of the area of Stochastic Programming, and to (37) for Stochastic Programming languages, environments, and applications. Interestingly, evolutionary approaches have not been applied to a wide range of real stochastic programming problems so far, with only a few examples available, e.g. recent works in the field of chemical batch processing, see (34) and (32).