# Upward Drawings of Trees on the Minimum Number of Layers
## (Extended Abstract)

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**Abstract.** In a planar straight-line drawing of a tree $T$ on $k$ layers, each vertex is placed on one of $k$ horizontal lines called layers and each edge is drawn as a straight-line segment. A planar straight-line drawing of a rooted tree $T$ on $k$ layers is called an upward drawing of $T$ on $k$ layers if, for each vertex $u$ of $T$, no child of $u$ is placed on a layer vertically above the layer on which $u$ has been placed. For a tree $T$ having pathwidth $h$, a linear-time algorithm is known that produces a planar straight-line drawing of $T$ on $\lceil 3h/2 \rceil$ layers. A necessary condition characterizing trees that admit planar straight-line drawings on $k$ layers for a given value of $k$ is also known. However, none of the known algorithms focuses on drawing a tree on the minimum number of layers. Moreover, although an upward drawing is the most useful visualization of a rooted tree, the known algorithms for drawing trees on $k$ layers do not focus on upward drawings. In this paper, we give a linear-time algorithm to compute the minimum number of layers required for an upward drawing of a given rooted tree $T$. If $T$ is not a rooted tree, then we can select a vertex $u$ of $T$ in linear time such that an upward drawing of $T$ rooted at $u$ would require the minimum number of layers among all other upward drawings of $T$ rooted at the vertices other than $u$. We also give a linear-time algorithm to obtain an upward drawing of a rooted tree $T$ on the minimum number of layers.

**Keywords:** Planar Drawing, Straight-line Drawing, $k$-layer Planar Drawing, Upward Drawing, Minimum Layer Upward Drawing, Trees, Algorithm, Line-labeling.

## 1 Introduction

A *$k$-layer planar drawing* of a tree $T$ is a planar drawing of $T$ where each vertex of $T$ is drawn as a point on one of $k$ horizontal lines called layers and each edge of $T$ is drawn as a straight-line segment. Such a drawing of a tree $T$ is also called a *planar straight-line drawing of $T$ on $k$ layers*. A tree $T$ is *$k$-layer planar* if $T$ admits a planar straight-line drawing on $k$ layers. For example, the tree $T_1$ in Fig. 1(a) is 1-layer planar since it admits a planar straight-line drawing $\Gamma_1$ on a single layer as illustrated in Fig. 1(b). However, an arbitrary tree $T$ may not always admit a $k$-layer planar drawing for a desired value

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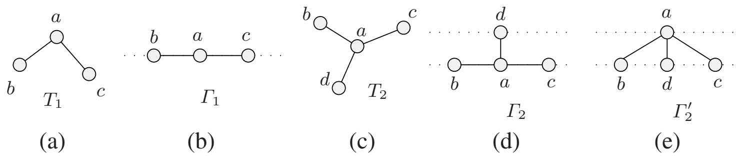


**Fig. 1.** (a) The tree $T_1$, (b) a drawing $\Gamma_1$ of $T_1$ on one layer, (c) the tree $T_2$, (d) a drawing $\Gamma_2$ of $T_2$ on two layers, and (e) another drawing $\Gamma_2'$ of $T_2$ on two layers

of $k$. For example, the tree $T_2$ in Fig. 1(c) does not admit a $k$-layer planar drawing for $k = 1$. The reason is as follows. Let $l$ denote the layer in the drawing on which the vertex $a$ of $T_2$ will be placed. If we want to obtain a planar drawing of $T_2$, then we can place at most two neighbors of $a$ in $T_2$ on the layer $l$. Placing all the three neighbors of $a$ in $T_2$ on the layer $l$ will violate planarity of the drawing and thus, at least two layers are necessary for a planar straight-line drawing of $T_2$. Two drawings $\Gamma_2$ and $\Gamma_2'$ of $T_2$ on two layers are shown in Fig. 1(d) and (e) respectively. Thus, although the tree $T_2$ admits a $k$-layer planar drawing for $k = 2$, it does not admit a $k$-layer planar drawing for $k = 1$. One can easily infer from this simple example that, such problems as to determine whether a given tree $T$ admits a $k$-layer planar drawing for a given value of $k$, or to compute the minimum number of layers required for a $k$-layer planar drawing of $T$ are quite challenging.

Let $T$ be a rooted tree. An *upward drawing of* $T$ *on* $k$ *layers* is a $k$-layer planar drawing of $T$ such that, for each vertex $u$ of $T$, no child of $u$ is placed on a layer vertically above the layer on which $u$ has been placed [6,7]. A *minimum layer upward drawing* of the rooted tree $T$ is an upward drawing of $T$ on the minimum number of layers. For example, if the vertex $a$ is taken as the root of the tree $T_2$ in Fig. 1(c), then the drawing $\Gamma_2$ of $T_2$ in Fig. 1(d) is a minimum layer upward drawing of $T_2$. If a tree $T$ is not a rooted tree, then let $T_u$ be the tree obtained from $T$ by considering a vertex $u$ of $T$ as the root of $T$. Let $v$ be a vertex of $T$. Let $l_v$ be the number of layers required for a minimum layer upward drawing of $T_v$. If for any other vertex $w$ of $T$, an upward drawing of $T_w$ requires at least $l_v$ layers, then a *minimum layer upward drawing of the tree* $T$ is an upward drawing of $T$ on $l_v$ layers.

A $k$-layer planar drawing of a tree is a common variant of the well-known “layered drawings” of trees [15,14]. In a *layered drawing* of a tree $T$, the vertices are drawn on a set of horizontal lines called layers, and the edges are drawn as straight-line segments [14]. Layered drawings have important applications in several areas like VLSI layouts [10], DNA-mapping [16] and information visualization [1]. Layered drawings of trees are usually required to satisfy some constraints arising from the application at hand. One such constraint is to impose bounds on the number of layers [14] and in this regard, it is often sought to know whether a given tree $T$ admits a $k$-layer planar drawing for a given value of $k$ [14]. However, the solutions for this problem known to date work only for some small values of $k$ [4,2,5]. For example, linear time algorithms have been given in [2,5] for recognizing and drawing trees that are $k$-layer planar for $k = 2$. For $k > 2$, Felsner *et al* [5] have given necessary conditions for a tree $T$ to be $k$-layer planar, but these conditions are not sufficient. For a tree $T$ with pathwidth $h$, a linear-time algorithm has been given in [13] to draw $T$ on $\lceil 3h/2 \rceil$ layers and it has been shown that $T$ cannot be drawn on less than $h$ layers [13]. However, the algorithm presented